AIR-TO-AIR COMBAT MODEL
DESCRIPTION & DEVELOPMENT
GENERAL INFORMATION

November 1967

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DEPUTY FOR DEVELOPMENT PLANNING
AERONAUTICAL SYSTEMS DIVISION
WRIGHT-PATTERSON AFB, OHIO

229

CAYWOOD-SCHILLER, ASSOCIATES

401 North Michigan Avenue

Chicago, Illinois

60611

ATAC-2: Single Search and Double Search

Terrell J. Harris
Barton J. Jacobs
Walter J. Strauss

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NOTE

This edition of the report on ATAC-2 is organized somewhat differently from the original edition. Volume I (Air-to-Air Combat Model, Description and Development, General Information) of the current edition consists of Volumes I and II of CSA Report Number 67-101 and of CSA Report Number 67-102. Volume II (Air-to-Air Combat Model, Program and Appendices, Technical Details) of the current edition consists of Volumes III and IV of CSA Report Number 67-101.

The Table of Contents and page numbers of the original edition are preserved here.

PREFACE

This report is published in four volumes. Volume I, *Model Description*, presents an overall view of the model and its two major submodels, the ENGAGEMENT Model and DATA PROCESSING Model. Volume II, *Model Development*, contains the rationale for the development and discussion of details, together with the derivations of all equations. Flow charts and program listings appear in Volume III, *Program*. Volume IV, *Appendices*, contains discussions of certain model concepts in detail.

The entire report is UNCLASSIFIED.

This report supersedes the original ATAC-2 document [Ref. 1]. The many changes and modifications made in the evolutionary development of the model, based on the analysis of many computer runs, have rendered the earlier version outdated. The program of the model as reported here was used for production runs in June, 1967.

Certain modifications which allow either aircraft to detect initially are reported separately in the document "Fighter Vs. Fighter Combat: ATAC-2 Model: Double Search," [Ref. 2].

ABSTRACT

>ATAC-2 is a simulation model designed to help evaluate fighters in air-to-air combat. The model treats the one vs. one dogfight which arises from a random search situation. Both aircraft in the combat are (usually) aggressive. The two principal outputs from the model are the probability a given aircraft is killed in the fight and the expected number of enemy aircraft an aircraft kills over its useful life. Combat is restricted to a fixed altitude. The maneuvers are dynamic in that each aircraft responds to the situation at each moment in a duel depending on the information it has about an opponent's activities.

Inputs include, for each aircraft, search and tracking radar characteristics, passive radar sensors, optical capability, IFF, energy-maneuverability data, weapon loadings, weapon characteristics, and weapon kill probabilities.

The rationale for the model specifics are presented. Flow charts and program listings are included. The model has been run repeatedly on an IBM 7094.

ACKNOWLEDGMENT

It is a pleasure to acknowledge the technical contribution of the personnel of the Directorate of Operations Research with whom and for whom this model was developed. In particular, it is noted that many helpful comments, suggestions, and criticisms were made by Lawrence Boyd, Duane Dunlap, Walter Grady, Thomas Haney, Lt. Kenneth Piersella, and Ralph Schwab.

It is also a pleasure to acknowledge the many suggestions and ideas of our colleagues at CSA. In particular, it is worthy to single out Thomas Caywood, James Cerone, James Hall, Thomas Morton, James Rust, Donald Schiller, and Ronald Swanson.

The model was programmed by Jeannette Filsinger and Fay Young of Deputy for the ~~Systems~~ Engineering ~~Group~~, ASD, at Wright-Patterson Air Force Base. Their patience and response to the frequent changes and modifications made in the flow charts are appreciated.

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SECTION 1

INTRODUCTION

ATAC-2 is the latest of a series of models developed to evaluate aircraft performance in Air-To-Air Combat, hence the acronym. ATAC-2 was designed principally to evaluate the outcome of a dogfight between fighters, although it can be used to evaluate the outcome of a fighter attacking a non-maneuvering, non-firing bomber. It starts with one aircraft detecting another at a random point. The aircraft that detects is predetermined. This characteristic of the model, that only one of the two aircraft is searching for the enemy, produces the title Fighter Vs. Fighter Single Search. Once detection occurs, the model traces through the ensuing sequence of events by means of a deterministic, time slice simulation. No Monte Carlo processes are involved. After the initial detection by one aircraft, the model treats the two aircraft alike, subject only to the constraints inputted for each aircraft. At any particular moment in time, an aircraft may be flying straight, be in a turn, or be on a form of pursuit course. It may be flying at constant speed, accelerating or decelerating. Both aircraft (usually) attempt to maneuver themselves into positions to fire their weapons. The model is dynamic in that at any given time the maneuvers performed, and the times at which weapons are fired by each aircraft, depend on the relative position of the aircraft. Further, the maneuver and weapon firings depend upon the information each aircraft has about the position and activity of the other aircraft at the time. The information made available to an aircraft depends on the sensors specified for it.

ATAC-2 consists of two major submodels: The ENGAGEMENT Model (EM) and the DATA PROCESSING Model (DPM). The EM answers questions such as: Can either aircraft fire its weapons? If yes, when, which weapona and how often? The EM produces a time ordered sequence of events that occur during a dogfight; the DPM transforms the sequence into overall battle outcomes such as the probability of kill of an aircraft.

A primary objective in designing ATAC-2 was to conserve running time so that a great many parametric variationa could be investigated with a reasonable amount of computer time. For this reason, the EM and DPM were designed to be run aeparately. While the running time of the EM is not excessive (much less than a minute on an IBM 7094 for a typical engagement), its running time per case is much longer than that of the DPM. The models are designed, therefore, so that some parametric variations can be investigated by running the DPM several times for one run of the EM. Such parametric variations include an aircraft's maximum combat time (i.e., fuel constraint), an aircraft's weapons configuration, and the kill probabilities attributed to each weapon type.

As an example of this parametric variation, a large variety of weapons might be input for an aircraft. The ENGAGEMENT Model would produce its time ordered sequence of firings of these weapons during the dogfight. The DPM could be run several times then, varying the set of weapons actually on board by ignoring certain firings. In order to achieve this feature, the EM was designed to ignore the effects of weapons fired. Outcomes of weapon firings are determined strictly by the DPM.

The major features of the models are discussed in the remainder of Section 1. This section concludes with an example of a run of the model. More detailed discussions of the EM and DPM are presented in

Volume II, Sections 4, 5, and 6. Definitions of the symbols which appear in each model are included in Section 7. Important restrictions on inputs to the models are presented in Section 7.3. Section 5 includes diagrams which are intended to clarify the definitions of certain symbols and a description of the conventions and symbols applying to the flow charts which appear in Volume III, Section 8.

1.1 MAJOR ASSUMPTIONS

(1) Combat takes place between only two opposing aircraft. This is a one-versus-one model.

(2) All maneuvers of both aircraft are confined to a horizontal plane.

(3) A maximum time can be imposed on the duration of an engagement, but this remains fixed regardless of the fuel consumed by an aircraft's acceleration or deceleration.

(4) Whatever information an aircraft hes about itself or the other aircraft is perfect. Thus, there is no false information on which en aircraft acts. An aircraft, however, may have incomplete information.

(5) An aircraft's action is not delayed by any reaction or response time. Thus, an aircraft can change its course instantaneously from a straight line to a maximum g turn.

(6) An engagement is initiated at the time one aircraft, designated "fighter," first detects the other aircraft, designated "bomber." Before being detected the bomber is presumed to be flying straight and level as if it were oblivious to the presence of the fighter.

(7) Prior to detecting the bomber, the fighter has no information on the position, heading or speed of the bomber. This model, therefore, reflects combat arising from a random search by the fighter.

(8) If the conditions for firing a weapon are satisfied the probability of kill by that weapon is independent of the geometry.

(9) There are no partial kills. A weapon either kills or does not kill an aircraft. An aircraft cannot be killed by demage compounded over several weapons.

(10) The time delay between the firing of a weapon and when it kills the target is a constant for all weapons of both aircraft.

(11) IFF has to be established only once for each aircraft.

(12) A bomber not already aware of the presence of the fighter will become aware if fired upon.

1.2 ENGAGEMENT Model

This model simulates a dogfight between two aircraft by examining the combat at fixed increments in time called "time pulses." Time pulses are introduced so that the position of each aircraft can be computed, the information available to each aircraft can be assessed, and each aircraft can react by changing its maneuvers and speed and by firing its weapons.

The general tactical doctrine of the EM for each aircraft is: Get close behind the target and stay there. Do this as soon as possible without exposure to the target's fire (the target's tail gun excepted). This assumes each aircraft has short range weapons with which to kill the target. The decisions each aircraft may make in each moment of time are:

1) speed up, slow down, or maintain speed;

2) turn left, or right, or fly straight;

3) turn on tracking radar;

4) fire weapon.

Both 1) and 2) are quantitative as well as qualitative and are dependent on each other.

The EM starts with detection. Both aircraft are assumed to have been flying straight before the EM begins, with an angle ε between their velocity vectors. For a given pair of velocity vectors detection of the bomber by the fighter can occur only if the bomber's path sweeps through the detection coverage of the fighter. If this condition is not met, combat will not take place. The probability that the condition is met is later calculated in the DPM. From the continuous contour of the fighter's detection coverage, along which detection may take place, several points are selected as starting points for combat. These starting points are also called "grid-points." Starting from a grid-point, the model steps through the paths of the combatants in time. Certain decision rules or tactical doctrines are built into the logic of the model. While the model provides some choices which would allow an aircraft to evade temporarily, to date evasion has not been exercised. Generally speaking, the decision rules which have been exercised reflect an aggressive attitude for each aircraft.

A narrative description of a possible engagement will serve to clarify the EM's operation. The fighter searches for the bomber. At some point in its path, the fighter stumbles upon the bomber. This is a grid-point at which a simulated engagement is initiated. Immediately the fighter speeds up and turns so as to get into a favorable firing position behind the bomber. As long as the bomber remains unaware, the fighter tries to maintain the surprise element. The fighter sneaks around to the rear of the bomber by skirting the latter's detection coverage. In order not to alert the bomber, the fighter holds fire until it gets to a preferred firing position, say 4,000 ft., from which it can launch a lethal salvo of weapons. Before firing, however, the fighter must identify the target by means of IFF and turn on its tracking radar. If the bomber becomes aware of the fighter before the fighter reaches the preferred firing position, the fighter realizes that the element of surprise is gone and will fire its weapons whenever possible. Either way the fighter continues to press the attack and strives to reach an advantageous position behind the bomber, say at 600 ft., from which it can fire its guns or other short range weapons. Given sufficient maneuverability, the fighter turns inside the bomber and stays at constant range and constant angle-off while firing short range weapons.

While the fighter attempts to do all this, the bomber in turn tries to attack the fighter with similar types of maneuvers. The bomber will do so as soon as it becomes aware of the fighter. Thus, except for the fact that the fighter initially surprises the bomber, the roles of the combatants are identical. Each tries to attack the other.

The success of the attack depends upon the ability of the attacker to fire its weapons. This depends on the range, the speeds of the combatants, their g-loading, the attacker's angle off the target, and the heading angle of the attacker relative to the target.

The maneuvers described above consist of turns and changes of speed consistent with the energy-maneuverability inputa for each aircraft. Geometrically these maneuvers may consist of scissoring, splits, pursuit courses, maximum g turns, etc.

The combat terminates if certain conditions are met. The model then recycles and simulates another engagement by starting with another initial position or grid-point at which the fighter detects the bomber. Several grid-points for a given pair of velocity vectors are located so that in the DPM, averages of the results can be calculated to represent the outcomes for engagement from this pair of initial velocity vectors and the angle ε. When the results for all desired grid-points have been computed, the model recycles for the next desired angle ε between the velocity vectors of the aircraft. Thus, the DPM can average over the angles ε as well to compute the probabilities of outcomes for a random engagement, or an encounter between two aircraft.

1.2.1 Initiation

ATAC-2 simulates combat that arises from random search by the fighter for the bomber. Initially the aircraft are assumed to fly straight and level at constant speeds at certain relative heading angles. If the fighter is equipped with detection radar, it searches with this only; otherwise, it searches optically. The model itself computes the arbitrary but fixed number of grid-points at which initial detection may take place. Detection of the bomber could not have occurred earlier.

Although the bomber might have detected the fighter before reaching the grid-point, it does not react until then. This rule was built into the model for purposes of simplification. It reflects the case in which one aircraft always surprises the other initially. In the ATAC-2 Double Search Mode, [Ref. 2] this restriction is removed.

1.2.2 Maneuvers

The allowed maneuvers, and the built-in decision processes which determine when each should be used, constitute the assumed tactics of air-to-air combat and as such are an important part of the model.

The basic tactic built into the ATAC-2 model is the attack by means of a decreasing lag pursuit course (DEL Pursuit Course) for each of the combatants. If an aircraft knows the whereabouts of the enemy but cannot at the moment fly this pursuit course, the aircraft usually changes its speed so as to maximize its turning rate and minimize the time necessary to establish a pursuit course.

When an aircraft reaches the proper heading for a DEL pursuit course, it presses the attack.

At any given time pulse, an aircraft follows one of three maneuvers: straight line, circle, or pursuit course. A bomber that has had no information about the presence of the fighter flies a straight line at constant speed.

An aircraft that has lost all information about the present whereabouts of its opponent changes its speed and turns at its best turning rate in an effort both to evade its opponent and to find it.

An aircraft obtains information about the whereabouts and maneuvers of its opponent only by means of the sensors. Each aircraft may be equipped with any combination of the following sensors: detection radar, a tracking radar, an optical capability, and one of two typea of passive radar sensors. These passive sensors may or may not be capable of recognizing the side on which an opponent is positioned. By means of the sensors the bomber may become "aware" of the fighter's proximity. "Aware" means that the bomber knows that a fighter is nearby. An aware bomber turns into the fighter seeking to attain a pursuit course if it can discern which way is "into." However, if the bomber's awareness is by means of a non-directional passive radar receiver, the bomber makes a tactical doctrine counterclockwise turn.

It is assumed that either the active radar or the optical capability is sufficient to maintain a pursuit course. While on a pursuit course, an aircraft accelerates or decelerates according to the doctrine that it arrive in a firing position for its shortest range weapon while moving at the same speed as the enemy (if possible), and that it should get to that firing position in minimum time. Thus, in general, the aircraft will accelerate until it must decelerate. If the aircraft is unable to maintain a pursuit course due to g loading, it reverts to a turn toward the enemy and adjusts its speed to improve its turning rate.

In all of the above statements, except the initial detection of the bomber by the fighter, the bomber and fighter are completely interchangeable.

1.2.3 Firing Conditions

Up to six different types of weapons may be included in an aircraft's configuration. (This is a program limitation, not a model restriction.) When an aircraft is within range of a target and certain conditions are met, it may fire its weapons at a predetermined rate. The allowable range interval in which a given weapon type may be fired depends on the geometry of the situation at the particular moment. It depends on the target's speed and g loading and on the attacker's angle off the target. In addition, it depends on the pursuer's speed, tracking angle and g loading. These factors together determine whether a launched weapon can hit the target. It is assumed that the conditions at time of launch (or firing) are sufficient to determine an intercept (or hit). The time of flight of the weapon is introduced in the DPM. The effect of the weapons on the target is superimposed on the combat in the DPM. This determination is deferred so that computer running time can be saved. Different combinations of weapons carried and different kill probabilities can be superimposed in the DPM on the same basic engagements without rerunning the simulation part (EM) of ATAC-2.

Firing occurs only after the attacker has established IFF by having had the target within IFF range and angular limitations.

The launch conditions may be different for each weapon. Thus, for example, the model can treat aircraft armed with Sparrow and Sidewinder missiles as well as guns.

The aircraft does not have to be on a pursuit course to fire; snap shooting is allowed.

1.2.4 Termination

An engagement ends when any of the following conditions obtain:

(1) the aircraft are within a minimum range of each other;

(2) the maximum combat time is exceeded;

(3) neither aircraft has information about the other, and an input amount of time has passed.

The minimum range in condition (1) can be input to zero. Condition (2) is put in to prevent the model from cycling through an endless loop, as for example, if two identical aircraft are circling about each other at identical speeds.

By "engagement" is meant the combat between two aircraft, starting with the detection of one by the other, continuing through the maneuvers and firings of weapons and terminating under the above conditions. Thus each simulation beginning with one grid-point for one ϵ is an "engagement".

1.3 DATA PROCESSING Model

The DPM is concerned with calculating the various conditional kill probabilities of interest. It uses the data produced by the EM which is already ordered by time for each engagement. The time-sequenced data of each engagement can be considered as a history of the combat between two aircraft. The DPM averages over many histories from many engagements, i.e., from many grid-points and many initial relative heading angles ϵ, to obtain the probabilities of kill of the encounter. It can use one run of the EM many times by varying the input P_K's, (probabilities of kill) or the type weapons, or the combat time available to each aircraft. It is for this reason that it was developed as a separate entity from the EM.

Each run of the DPM uses the results of one run of the EM. A run calculates the results for one specified pair of aircraft with specified initial speeds, weapons, etc., but for a whole set of grid-points and heading angles ε.

The DPM uses the time each weapon is fired. It assumes the P_K of a weapon is independent of the particular circumstances under which it is launched, such as range and angle. If this assumption is not a good one, then launch conditions which give rise to vastly different P_K's should be treated as though associated with a different weapon type in inputting the EM.

Many of the outputs of the DPM are in a form required as inputs to the AIR-SU-5 Model of AFGOA for force evaluation.

1.4 INPUTS

The inputs to the model involve aircraft characteristics, sensor properties, and weapon parameters. Other quantities which must be input are classified as miscellaneous. This class includes quantities which govern the initial and terminal conditions of simulated engagements.

1.4.1 Aircraft Characteristics

An aircraft is specified for the EM by values inputted for the following performance characteristics and conditions:

(1) initial speed,
(2) minimum speed,
(3) maximum speed,
(4) energy-maneuverability: specific power as a function of speed and g's,

(5) a constant factor for deceleration, aside from (4),
(6) maximum sustained g's which can be pulled in a constant speed turn as a function of speed, (related to (4)), and
(7) maximum g's a pilot can withstand.

An available combat time for each aircraft must be specified for operation of the DPM.

The designations of the combatants as fighter and bomber are used strictly for convenience in describing the operation of ATAC-2. They are not intended to imply any restrictions about the type of aircraft which can occupy the roles of either combatant. The user is at complete liberty to specify the type of aircraft for the role of fighter or bomber and can, if he wishes, interchange the roles of aircraft types in successive runs of ATAC-2.

1.4.2 Sensors

ATAC-2 allows five sensors to be included in an aircraft's configuration:

(1) detection radar,
(2) tracking radar,
(3) IFF,
(4) passive radar receiver, and
(5) optical.

The coverage patterns of all sensors are assumed to be sectors of circles. A range and half-angle must be specified for each sensor to be included in an aircraft's configuration. The range of any of the five possible sensors which are not to be included must be set to zero. All half-angles are measured with respect to the nose of an aircraft except the half-angle of passive equipment which is measured with respect to an aircraft's tail.

In addition to range and half-angle, the type of passive radar receiver to be included in an aircraft's configuration must be specified. ATAC-2 considers two classifications of passive equipment:

(1) a type with side discrimination, and

(2) one without side discrimination.

Passive equipment with side discrimination is capable of distinguishing the side, left or right, from which an attacker is approaching.

In ATAC-2, a prerequisite for passive detection to occur is that the detecting aircraft must be illuminated by the other combatant's tracking radar. Illumination by detection radar is not considered sufficient for passive detection because it is presumed to involve an unacceptably high false alarm rate.

The coverage specified for tracking radar equipment should be included within the coverage of an aircraft's detection radar. If this restriction is not observed, an aircraft may be prevented from firing when it logically should be able to, since in the model the tracking radar is insufficient to cause firing.

ATAC-2 allows a weapon to be fired only if a target aircraft is, or was, at least once, positioned within an attacker's IFF coverage. Thus, some IFF coverage must be specified for an aircraft if the user wishes to allow the aircraft to fire its weapons.

The active detection capability of an aircraft designated the fighter serves a special function in initiating simulated engagements. The bomber occupies a position on the perimeter of the fighter's active detection pattern at the beginning of each engagement. The active detection capability used for this function is that of the fighter's detection radar, if the fighter has detection radar; otherwise, the fighter's optical capability is used.

1.4.3 Weapons

Up to six weapon types may be included in each aircraft's configuration. For each weapon type, launch envelopes or firing tables must be provided which may be sensitive to a target's speed and turning rate and the attacker's angle off the target aircraft's tail. These tables specify minimum and maximum ranges within which an attacker may successfully fire a weapon of a given type. Further inputs are the maximum allowable tracking angles and g limitations which must be satisfied for an attacker to fire each of his weapons. For the DPM, the probability of kill given a weapon is fired must be provided for each weapon type. If weapons of a given type are to be fired in salvos, the probabilities of kill input should be for a salvo. A salvo is treated as ne weapon firing. The minimum time between successive firings of a given type weapon is an input for each type. The EM provides most of the inputs needed by the DPM. If desired, the DPM utilizes only a subset of the weapons carried by the aircraft in the duel in the EM. Each weapon in all such subsets to be considered must be specified in the inputs for the DPM. The EM prints out the conditions under which each weapon was fired.

1.4.4 Miscellaneous

The miscellaneous category of inputs refers to control parameters which govern the operation of the ATAC-2 computer program. The principal inputs in this category are:

(1) Δt , the length of a time pulse. The EM evaluates engagements between two aircraft at successive intervals of time called time pulses. Aircraft are moved relative to each other, weapon firing conditions are examined and decisions about the next aircraft maneuvers to be executed are made in each time pulse. The accuracy of the outputs of the model as well as the computer time required for the execution of engagements depends upon the value assigned to Δt .

(2) The maximum time for an engagement. An engagement is terminated under a number of conditions. One of these is the expiration of the maximum simulated time allowed for an engagement. The value of this input should be a reflection of the combat time and, therefore, the fuel supply and fuel consumption rate of the aircraft being considered.

(3) Minimum range between aircraft. Another condition for terminating an engagement is the range between the two aircraft becoming less than a specified minimum. This parameter may be set to zero, thus eliminating the condition.

(4) $\Delta\varepsilon$, the crossing angle increment. ATAC-2 provides for consideration of a series of initial relative headings of the two combatants for a given run of the computer program. The relative heading at the beginning of each engagement is given by ε , the angle between the fighter and bomber initial velocity vectors. The program will consider a set of ε's uniformly spaced by the increment $\Delta\varepsilon$. For details, see Volume II.

(5) The number of points, called grid-points, on the perimeter of the fighter's active detection pattern at which the bomber will be initially positioned for a series of engagements for each ε . The number of grid-points, together with the number of ε's which is determined by $\Delta\varepsilon$, affect the reliability of the kill probabilities which result from the averaging processes used by the DPM.

(6) A choice of evasion options. Under several conditions, such as running out of ammunition, or having inferior relative turning ability coupled with a disadvantageous position, an aircraft may try to evade its opponent. However, to date, these options have not been exercised.

1.5 Output of ENGAGEMENT Model

One run of the EM is executed for two specific aircraft with input initial speeds, weapons, etc. A run includes many engagements by looping through several grid-points for each ε and several ε's . Typically one run of 50 engagements (5 ε's , each with 10 grid-points), corresponding to 5 minutes of combat time each, takes 10 minutes on an IBM 7094. Thus, each engagement runs in about one-twenty-fifth of real time.

As an optional output for each engagement, the program prints the inertial coordinates of the position of each aircraft at regular intervals of time. This allows one to trace the time-linked paths of the aircraft in space.

Regular outputs for each engagement are:

(1) For each weapon type of each aircraft, the times at which that aircraft launches those weapons (if at all), the range and relative angles at launch and the cumulative time during which the launch conditions for that weapon type are met. The amount of ammunition fired is thus directly considered. By a weapon "launch" is meant the firing of a weapon when the required launch conditions obtain.

(2) The time at which the bomber becomes aware of the fighter.

(3) The total time of the engagement.

(4) The relative angle ε and the grid-point of the engagement.

Regular outputs for each run include the necessary identification information.

1.6 Output of the DATA PROCESSING Model

Most of the outputs of the DPM are probabilities. These probabilities are generated by first calculating the corresponding probabilities for each engagement, i.e., for each grid-point of a given ε, then averaging over the grid-points and finally over the ε's. The probabilities of kill for an engagement are computed by restricting combat to the available combat time of the aircraft.

Typically, one run of the DPM, corresponding to one run of the EM, takes about 10 seconds on the 7094.

Most of the probabilities computed are conditional probabilities. This means that the events for which the probabilities are computed depend on other events occurring. Over all, it is noted that the EM limits events to those in which the fighter detects the bomber. To remove this condition, the DPM must evaluate the probability of the event that the bomber is detected.

Given detection by the fighter, the bomber may or may not be aware of this detection. "Aware" means that at some time during an engagement the bomber detects the fighter; "unaware" means that the bomber never detects the fighter during a given engagement. Another conditional output computed is the probability of the event that the bomber is killed given it is detected and is unaware. The DPM uses such output to find the joint probability of kill and unawareness, by evaluating the probability that the bomber is unaware given it is detected. The complementary condition is sometimes imposed and outputted: that the bomber is aware. The DPM outputs the probability of the event that the bomber is killed given that the bomber is detected and is aware. Similarly, the DPM outputs the probability that the fighter is killed given that the bomber is detected and is aware. A further condition on the events for which probabilities are calculated is that a given aircraft does not fire. One corresponding output is the probability of the event that the bomber is killed given it is detected, aware and does not fire.

In general, when conditional probabilities are calculated, the probability of the conditions themselves must be evaluated to determine the unconditional probabilities.

Another set of outputs shows the expected number of targets an aircraft kills over its useful life. This is a measure of effectiveness of an aircraft. It takes into account both its ability to kill a target and its ability to survive an encounter with an enemy.

1.7 An Example of an ATAC-2 Run

A tool of unquestionable value in the development and understanding of the ATAC-2 Model is the plot of the coordinates of the combatants as a function of time. In Figure 1.7-1 a (partial) plot of an actual ATAC-2 run is shown. The time (in seconds) associated with the points along each curve is shown; the bomber's time is shown to the right of each marked point on its path, and the fighter's time is shown to the left of the corresponding point on the fighter's path.

In this figure the aircraft were initially about 4,000 ft. apart when the fighter detected the bomber. The bomber was initially flying at 750 ft/sec, and the fighter at 950 ft/sec. Immediately the bomber became aware of the fighter and turned hard left to acquire it. The fighter turned left to try to maintain the pursuit course. In so doing the fighter crossed the bomber's path, yet still remained behind it and did not lose the advantage. However, due to this crossing, or "overshoot," the bomber started to turn the other way, causing more overshooting of the fighter and producing a series of scissoring maneuvers by the two aircraft (times 11 to 29). Eventually the bomber, with its inferior position and inferior maneuverability, fell into a situation in which all it could do was to remain on a maximum g turn at its best turning rate, only to find the fighter could out turn it and maintein a position behind the bomber.

Although merely one example of the results obtainable from ATAC-2, this figure shows some interesting aspects of the model. For example, the bomber was able to know which side the fighter was on even when the fighter was in the bomber's rear. This is because the fighter's tracking radar was illuminating the passive sensors of the bomber, giving the bomber the hemisphere of the source and hence the direction in which to turn. However,

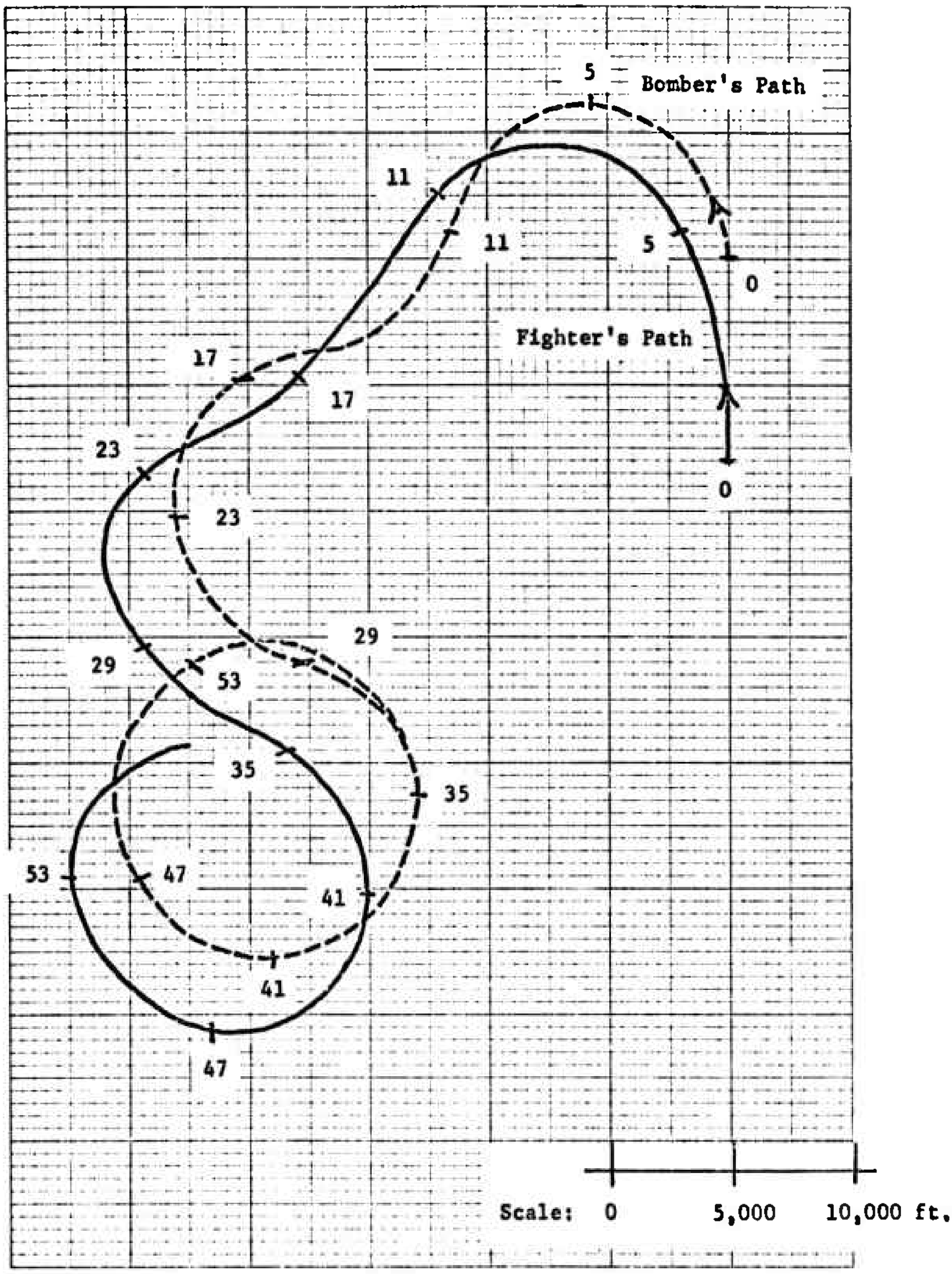


Figure 1.7-1 Plot of Engagement

later on (between points 23 to 29) the bomber lost this information due to a high angle-off of the fighter, and turned steadily away from the fighter. This in fact was its undoing. Also, throughout this example each aircraft varied its speed. Since the ability to pull g's is a function of speed, the g's pulled varied also. This is evidenced by the varying radii of curvature throughout the paths.

The outcome of this engagement depended on the weapon loadings of the aircraft, which could be varied in the DPM, but the fighter in any event would likely kill the bomber with high probability, since it consistently held good firing positions.

SECTION 2

MODEL CHARACTERISTICS AND ASSUMPTIONS

A checklist of the model characteristics and assumptions is presented here for the reader to scan, in order to emphasize just what the model is and does.

1. ATAC-2 is a computer simulation.
2. One aircraft versus one aircraft only.
3. All action is on a horizontal plane.
4. ATAC-2 is separated into the ENGAGEMENT Model (EM) and the DATA PROCESSING Model (DPM). The EM is the actual simulation.
5. Lethal effects of weapons are treated in the DPM but ignored in the EM, in which the two aircraft continue to fly.
6. The DPM uses an iterative procedure to calculate kill probabilities from the EM.
7. Basic tactics are built into the model; except for certain evasion options, inputs affect only details of tactics.
8. The aircraft designated F always detects the aircraft designated B first.
9. The position of B, before F detects, is a uniformly distributed random variable over an area.
10. Based on 9., the DPM computes kill probabilities independent of the position when detection occurred.
11. No Monte Carlo processes are used.
12. The final output of the model is, for each aircraft, the number of enemy aircraft killed over an aircraft's useful life. This measure combines an aircraft's ability to kill the enemy with the measure of its own survivability.

SECTION 3

POINTS OF EMPHASIS IN ATAC-2

As ATAC-2 developed certain areas were felt to merit more detail than others. A short summary of these points of emphasis is presented for introduction to the model discussion. They will be developed as needed in Section 4.

Above all, the model looks at tactical maneuvers more closely than at such considerations as weaponry, avionics, or overall mission success. In particular, aggressive maneuvers, as opposed to evasive, are considered in detail. The basic aggressive maneuver is the decreasing lag pursuit course (DEL Course), whereby a pursuer attempts to swing behind its target while pointing behind the target. One advantage of this course is that the pursuer quickly establishes a position well to the rear of the target, thus possibly obtaining an element of surprise. This surprise element will be considered in some detail in the model. For example, the pursuer will avoid alerting the target by not firing until within a range R^*.

Once the aircraft are close to each other, a major concept is the steady state. A pursuer attempts to maneuver so that the target is helpless if the target attempts to convert on the pursuer. In steady state, even while the target turns, the pursuer can turn at the same rate and stay behind the target.

This steady state is useful only at a good firing range. The pursuer tries to achieve the steady state at an ideal firing range R^*.

The ability to maneuver is based on linear acceleration and turning rate capability. This is defined by the specific power of an aircraft at a given power setting. The specific power function describes the trade-offs between accelerating and turning. Decisions must be made concerning these trade-offs. Although for a given velocity the turning rate determines the number of g's an aircraft pulls, and vice-versa, there is a subtle difference between making the decisions based on turning rate and based on g's. <u>Turning rate is thought to merit more attention than g's.</u> In particular, the optimal point in the trade-off between acceleration and turning rate frequently is the point at which the <u>largest sustained turning rate is achieved</u>.

The doctrine concerning speed in the model is that an aircraft should <u>speed up as much as possible until it must slow down</u> in order to reach a preferred firing position as fast as possible. A function <u>S(V')</u> is defined in the model to approximate the point where an aircraft must switch from acceleration to deceleration.

The ability to fire weapons plays the second most important role in the model. The major criterion for a weapon firing is that the launching aircraft be inside a <u>"weapon envelope"</u> surrounding the target. This envelope varies with the individual weapon, the target's velocity and maneuver. Some aspects of the envelope are input, others calculated dynamically.

Many other criteria are introduced in the model for weapon firings, and other considerations are put forward for tactical decisions. But the above underlined points pervade the model and are most basic to it.

SECTION 4

ENGAGEMENT MODEL DEVELOPMENT

4.1 Introduction

Consider two aircraft, each of which flies straight and level. One is a fighter searching for enemy bombers. The other is a bomber unaware of an impending air-to-air encounter. If the fighter detects the bomber, it engages the bomber in a duel. Either aircraft may then be shot down. The ENGAGEMENT Model mathematically formulates the search and the ensuing engagement.

The engagement can be broken down into segments, each of which can be described mathematically. For example, one can modelize the conditions for firing a weapon by an "envelope" which surrounds the target aircraft. One can then decide if a pursuer is inside this envelope. If so, and if certain other conditions are met, it fires; otherwise it does not (or at least it fails to hit the target).

Similarly, if the pursuer flies a pure pursuit course, which means it always points at ths target, then one can describe the position of the two aircraft after, say, 10 seconds. Severe restrictions, however, must be placed on the target's course (constant speed and direction) and the pursuer's speed (constant). But targsts do turn; to deny this would bias the model heavily to the pursuer's gain. So one must allow the target to turn. On similar grounds the aircraft must be allowed to accelerate. Each of the concepts of direction and speed change can be cast into a mathematical form. But it is hopeless to combine these expressions with all the other necessary equations in order to obtain a unified analytical expression which reflects air-to-air combat. An approach using simulation is essential. The simulation unifies these mathematical descriptions by incrementing time in discrete jumps (typically 1/4 second is small enough for accuracy). At each time pulse, decisions can be made about acceleration, weapon firings, etc. The rules of decision are inflexible during the engagement, but each decision is made on a cross-that-bridge-when-it-comes basis. This allows greater flexibility in the model. New decision rules and new concepts can readily be added. For example, if additional requirements were necessary for weapon firing, these could be inserted and interrogated before firing. In a closed form mathematical model this would not be easy.

The flow of ATAC-2 is shown by Figure 4.1-1; the time simulation is step three. This phase contains the actual engagement. Step two describes the fighter's search and eventual detection. The idea of the model flow is that for one input-set a number of engagements will be simulated. The difference between each engagement will be only the positions of the aircraft at initiation, defined as the point at which the fighter detects the bomber. The tactical rules and weapon loadings will be identical. Nevertheless, different starting points can lead to vastly different results.

4.1.1 Layout of Development

The model discussion centers attention first on the search of the fighter, which leads to possible positions where detection of the bomber occurs. Once the engagement begins, the first consideration is mechanical; a coordinate system must be set up, important geometric aspects considered, and basic equations derived describing the movement of the aircraft. From there the discussion proceeds to the heart of the model: the conditions for firing weapons, and the tactical decision rules. The former must be discussed

INPUTS

↓

GRID: Generate grid of possible initial positions from which aircraft designated F can first detect aircraft designated B .

↓

ENGAGEMENT: For each initial position simulate subsequent combat.

↓

PK: Calculate kill probabilities of each aircraft for each engagement. Combine these probabilities to arrive at P_K unconditional on initial position.

Figure 4.1-1 ATAC-2 MASTER FLOW

first as they give rise to the latter. In particular, requirements of weapons, along with other considerations, lead to the course taken by a pursuer in closing in on its target.

After describing this course, attention turns to the method by which the aircraft achieves this course. The aircraft changes its acceleration and turning rate constantly, and the mathematical description of this change is formulated. There are many restrictions on acceleration and turning, and decision rules are developed to take care of cases in which the pilot cannot achieve the doctrine course.

Finally Section 4.7 discusses tactics in more detail. For example, once the pursuer is close in on the target's tail, what does it do? (It must be emphasized that ATAC-2 postpones the question of aircraft kill until the DPM is exercised. In the ENGAGEMENT Model, the attack must proceed after weapon firings.)

Other points covered by Section 4.7 include the tactics of an aircraft when its radar or optical information is inadequate to follow the doctrine pursuit course. Also, an evasion option, as opposed to attack, is offered the aircraft.

4.1.2 Symbols

A list of all symbols is given in Section 7 of this volume. Symbols are consistent throughout. However, the evolutionary development of ATAC-2 resulted in certain unfortunate notations. The reader is cautioned not to associate a variable in simple form with one in subscripted form. For example, S , S_i , $S_i(x)$, $S(x, y)$ can each be a distinct variable with no relation to the others. Further, although rarely occurring, it can happen that the same variabla is used in different routines for different purposes. This cannot happen within the same routine, however.

4.1.3 Coordinate Systems

In both steps 2 and 3 of Figure 4.1-1, the aircraft are assumed to fly in tha same horizontal plane. This is not "realism," of course, but the ability to climb will be reflected in horizontal acceleration and turning. A two dimensional scheme reduces complexity, and thus increases understanding of tha model. For the two dimensions a rectangular (x, y) coordinate system is superimposed over the space the aircraft cover. (x, y) describes the positions of the aircraft. Tha time derivatives $\dot{x}$ and $\dot{y}$ of each aircraft describe the valocities. This (x, y) coordinate system is introduced at tha beginning of an engagement and remains fixed with respect to the ground; both aircraft change their (x, y) coordinates in time.

Two other dimensional systems are used. To describe the search phase, a moving rectangular coordinate system centered at the fighter is convenient, referred to as (X, Y). Then, during the engagement, the main concerns of tactics and weapon firings dictate a moving coordinate system in addition to (x, y). These relative coordinates are not rectangular but are defined by the range between the aircraft and relevant angles. Both aircraft take into consideration the relative geometry in their tactics.

Appendix B, Geometric Considerations, discusses some interesting aspects of the coordinate systems.

4.2 Search

Significantly, the time simulation begins at the meeting of tha aircraft. To evaluate the search phase, rather than fly the aircraft until tha fighter detects the bomber, it is usaful to consider what the model requires from the search phase.

First it requires a point of detection: The position of both aircraft at tha tima the bomber enters the detection coverage region of the fighter. The positions previous to this are of no consequence. Second, it requires a probability: How likely is this position given detection occurs? The model will exercise the engagement simulation a number of times for different detection positions, in order to arrive at results independent of the directions the two aircraft flew prior to the engagement. Probabilities ara needed to combine the results of the individual engagements. The search problem, then, is reduced to one point in time, the time at which detection occurs; thus no simulation is necessary. To analyze the search, conaider the fighter's detection equipmant. It is either radar or optical. Assume that if the aircraft has radar, it is used exclusively. Once detection occurs, however, the fighter will use both for continued use. With either means of detection, the area of coverage is described by a circular sector (actually a cone reduced to two dimensions). Figure 4.2-1 shows a typical radar pattern; 4.2-2 an optical. Normally, active radar is limited to the front hemisphere, while optical sighting allows the pilot to look behind, as in the diagram. In any case the fighter may always detect directly in front, out to a maximum range r, and on either side through an angle ρ. Along the sides of the detection pattern it can also detect out to r.

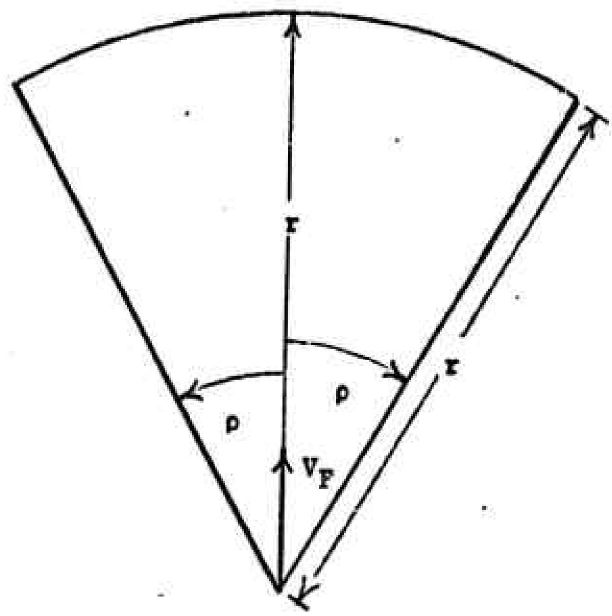


Figure 4.2-1 Typical Radar Pattern

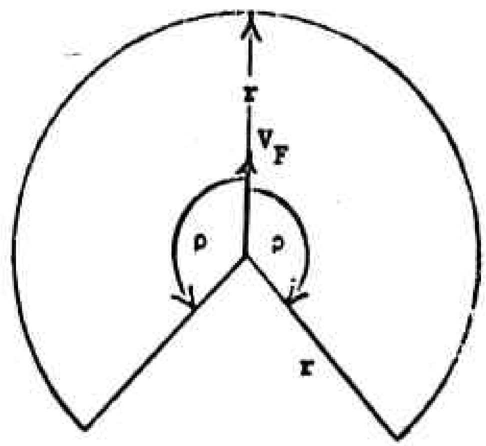


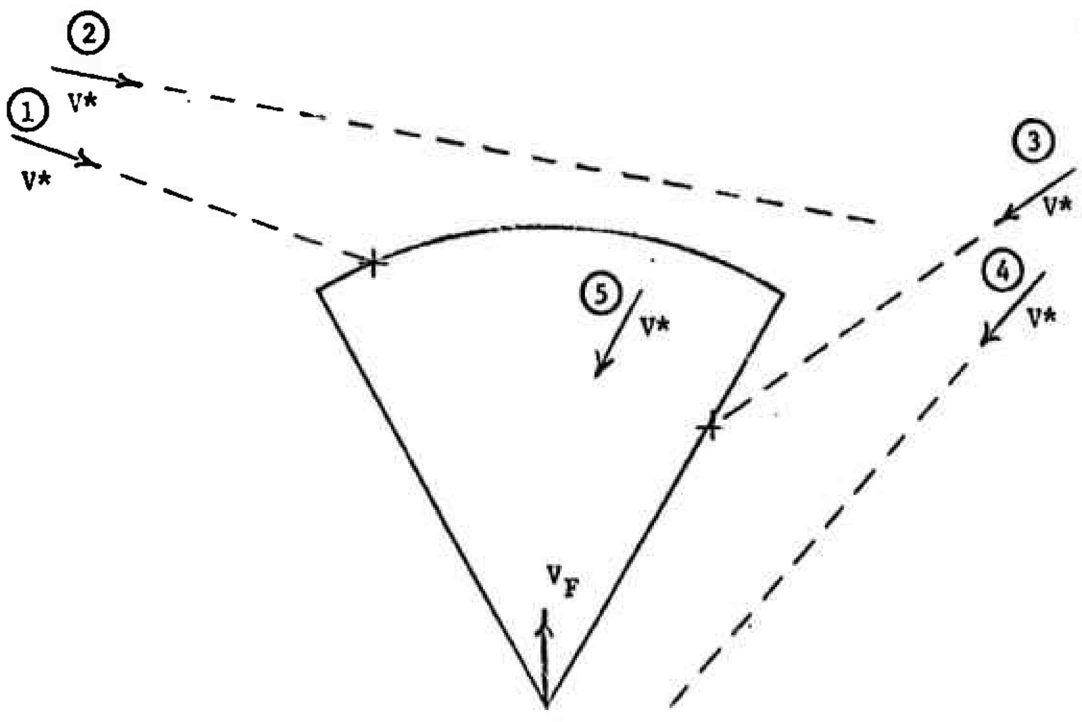
Figure 4.2-2 Typical Optical Pattern

The bomber is assumed to possess similar detection equipment, usually with a different range and angular limit. However, if the bomber detects the fighter before the fighter detects, the bomber will ignore it and fly straight until it sees the fighter maneuver. The bomber prefers to avoid encounter, and the fighter possibly will not detect.

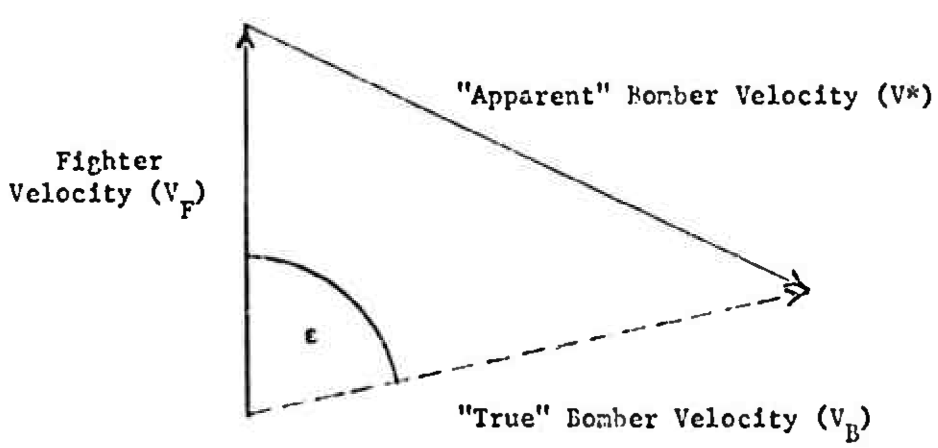
The bomber and the fighter fly linearly during the search. The fighter's detection pattern moves along with the fighter. To determine whether the bomber crosses this pattern, the fighter is "fixed"; that is, an (X, Y) coordinate system is established which moves with the fighter, and the bomber's change in coordinates will represent the change in relative position due to the motion of both aircraft. Figure 4.2-3 shows the fighter, its detection pattern, and several possible bomber positions. Note that the arrows showing the relative bomber velocity $\vec{V}*$ are not the velocities as a ground observer sees them; since the fighter is fixed, not the ground, the arrows describe the bomber velocity relative to the fighter -- how the fighter would see the bomber if it were inside its detection pattern. Thus, the angle between the two aircraft headings is not the angle between $\vec{V}_F$ and $\vec{V}*$. However, a simple relationship exists here as shown in Figure 4.2-3. The vector sum of the fighter velocity and the relative bomber velocity is the true bomber velocity.

Another point to note about V is that by reversing the direction of the arrow, the bomber can be considered fixed, and $\vec{V}*$ will represent the relative motion of the fighter.

Figure 4.2-3 shows the difficulty of handling some arbitrary bomber velocities. The bombers labeled 1 and 3 will eventually be detected, but 2 and 4 will not. 5 has already been detected.



Position Diagram



Velocity Diagram

Figure 4.2-3 Bombers in Random Directions

The geometry of each case is somewhat different. However, a uniform criterion arises if, as in Figure 4.2-4, the course of the bomber relative to the fighter is restricted to one direction. If relative to the fighter, the bomber flies within the segment Y^* it will be detected; otherwise not. If the fighter searches for time t, then relative to the bomber it moves a distance V^*t. The detection pattern relative to the bomber sweeps out an area $(V^*t)Y^*$. If the total area of possible positions for the bomber is known, then the probability of detection P_D is found by

$$P_D = \frac{V^*Y^*t}{\text{Total Area}} .$$

More generally, for a given ε, which is the angle between the bomber and fighter velocities $\vec{V}_B$ and $\vec{V}_F$, assuming a constant total area and search time,

$$P_D = P_D(\vec{V}_B , \vec{V}_F , \varepsilon , \rho , r) .$$

A more complete development is given in Appendix A.

The problem with tackling the general case of Figure 4.2-3 is now evident. Y^* will change with ε. Note that from Figure 4.2-5, if the fighter flies very fast, a tradeoff may occur. It covers more distance (V^*t) but Y^* shrinks.

Figure 4.2-6 shows the separation of the search problem into

1. picking an ε, and
2. considering possible detecting positions for the ε.

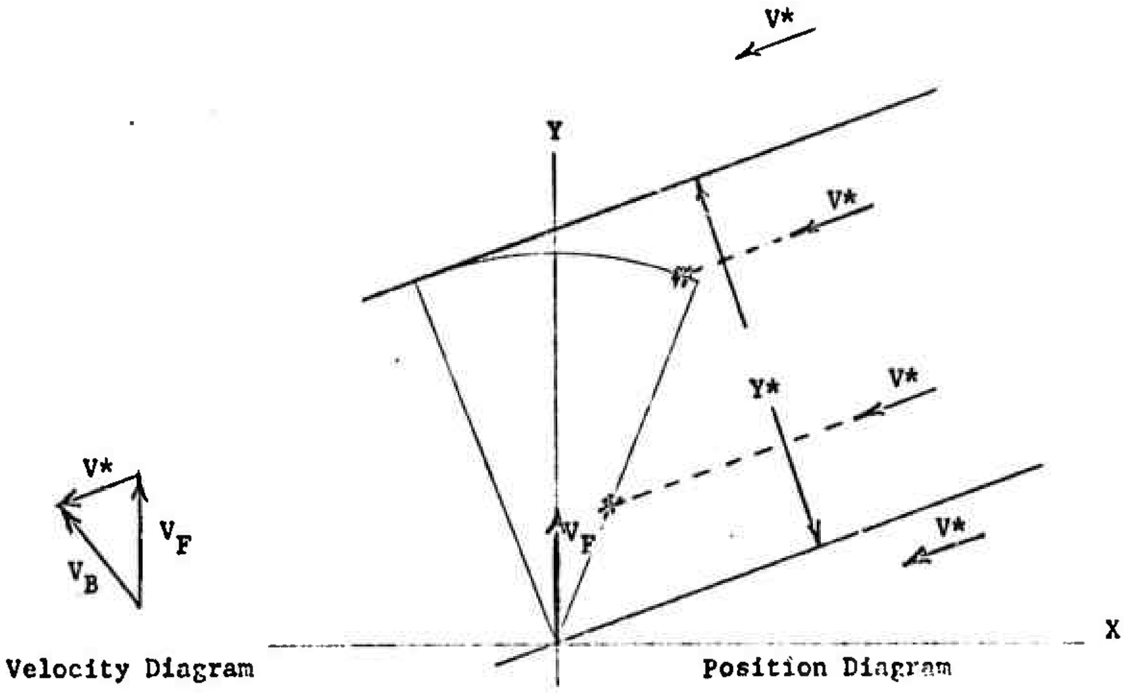


Figure 4.2-4 Bombers in Same Direction

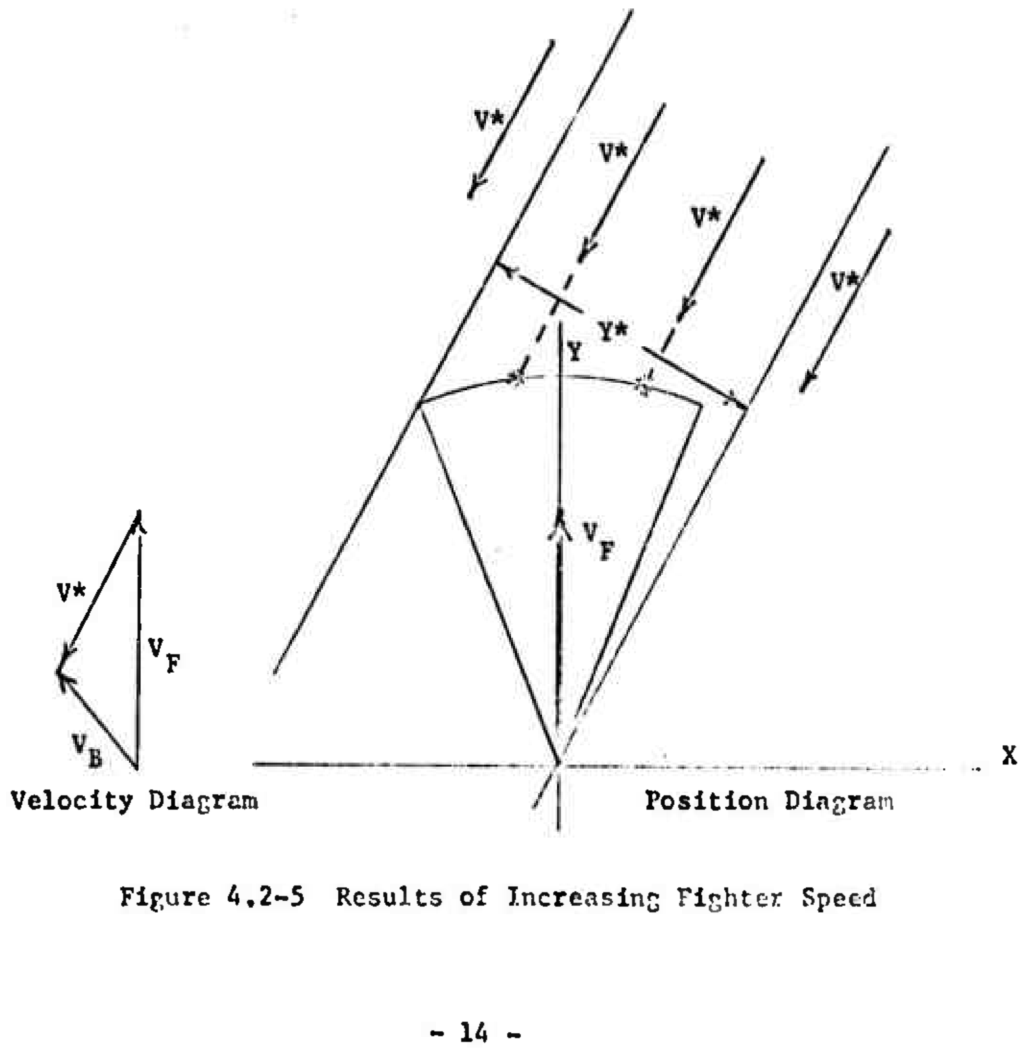


Figure 4.2-5 Results of Increasing Fighter Speed

<u>ε GRID</u>: Set up grid of "ε's," the angle between the two velocity vectors of the aircraft.

↓

<u>GRID FOR GIVEN ε</u>: For each ε, consider the fighter's detection geometry. Together they give rise to a set of possible points where the bomber could first come into view. Set up a grid of these points.

↓

OUT

Figure 4.2-6 <u>Search Flow</u>

From Figure 4.2-7 Y^* is obtained. For convenience, in the equations which determine the initial positions, the (X, Y) coordinate system is rotated so that the relative velocity vector V^* is parallel to the X-axis. The fighter is fixed. Now all that remains is to select a Y value, say Y_g, between Y_{min} and Y_{max}, which gives rise to a particular point of detection. Figure 4.2-7 is the normal case: The fighter initially sees the bomber from the front. Figure 4.2-8 shows a case in which the bomber is faster and comes from behind. In Figure 4.2-9 the situation is similar, but now the detection pattern allows the fighter's detection from the rear.

Figure 4.2-10 describes the flow of this model in a more procedural manner. In particular, it shows that Y_B and Y_C must be evaluated, the limits which separate the paths on which detection occurs at full range from paths on which detection occurs from the side (note Figure 4.2-9).

The model, then, considers a representative set of ε, and for each ε a representative set of Y along Y^*, and simulates the engagement from that point. The likelihood of each engagement can be evaluated, as was shown, and probabilities unconditional on ε are thus obtained.

4.3 Engagement Geometry

During an engagement, tactics are determined by certain geometric relations between the aircraft. The relevant geometry is shown in Figure 4.3-1. In the remainder of the discussion, the subscripts B and F will sometimes be dropped when the discussion applies to either.

The angle α between the velocity vector of the attacker and the line of sight is referred to as the tracking angle. The angle ϕ between

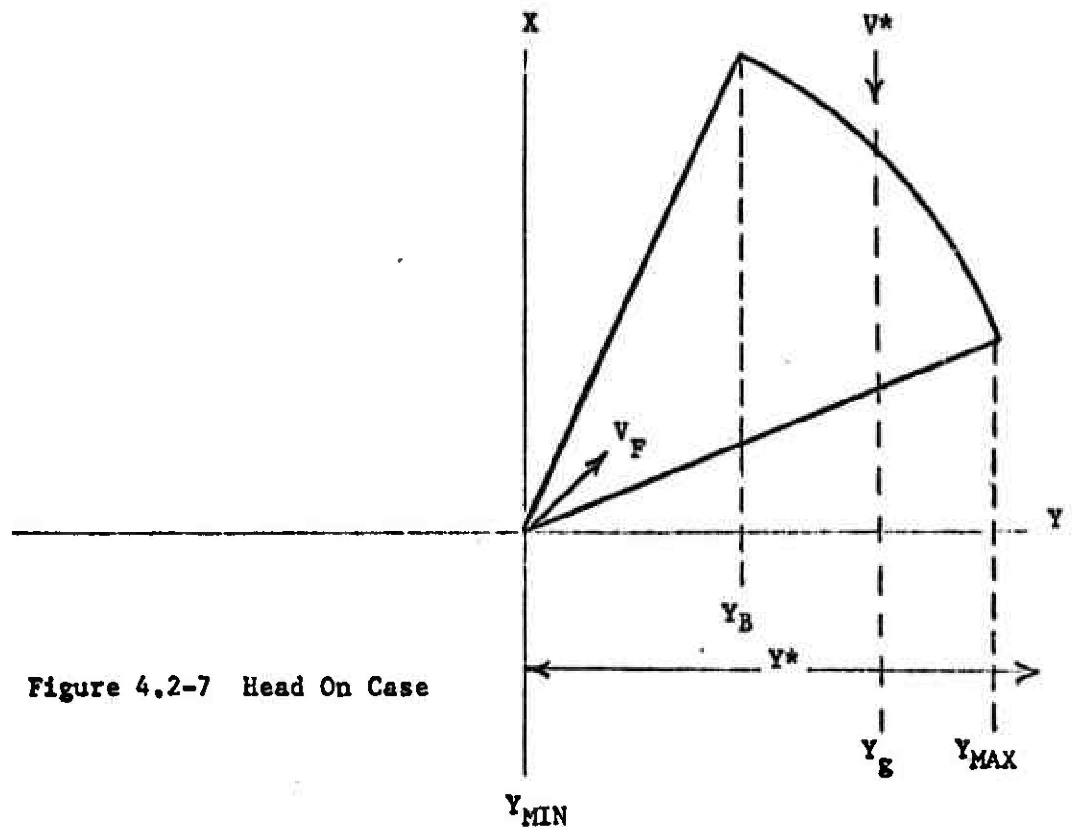


Figure 4.2-7 Head On Case

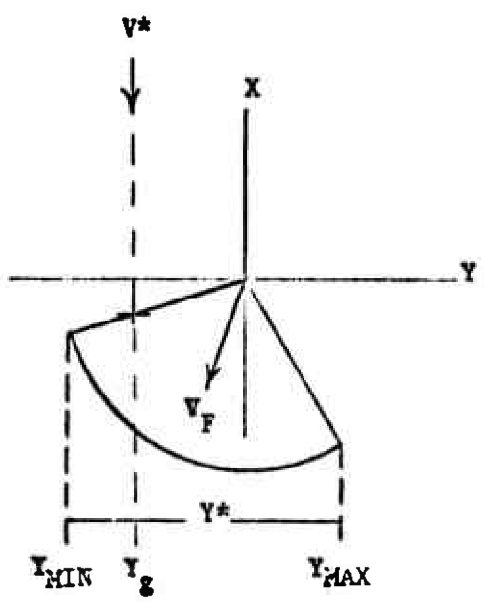


Figure 4.2-8 Bomber Comes In From Behind

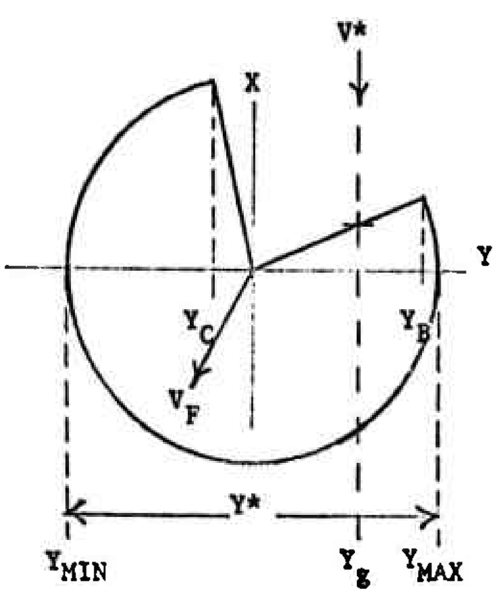


Figure 4.2-9 Fighter Detects While Bomber is in Rear

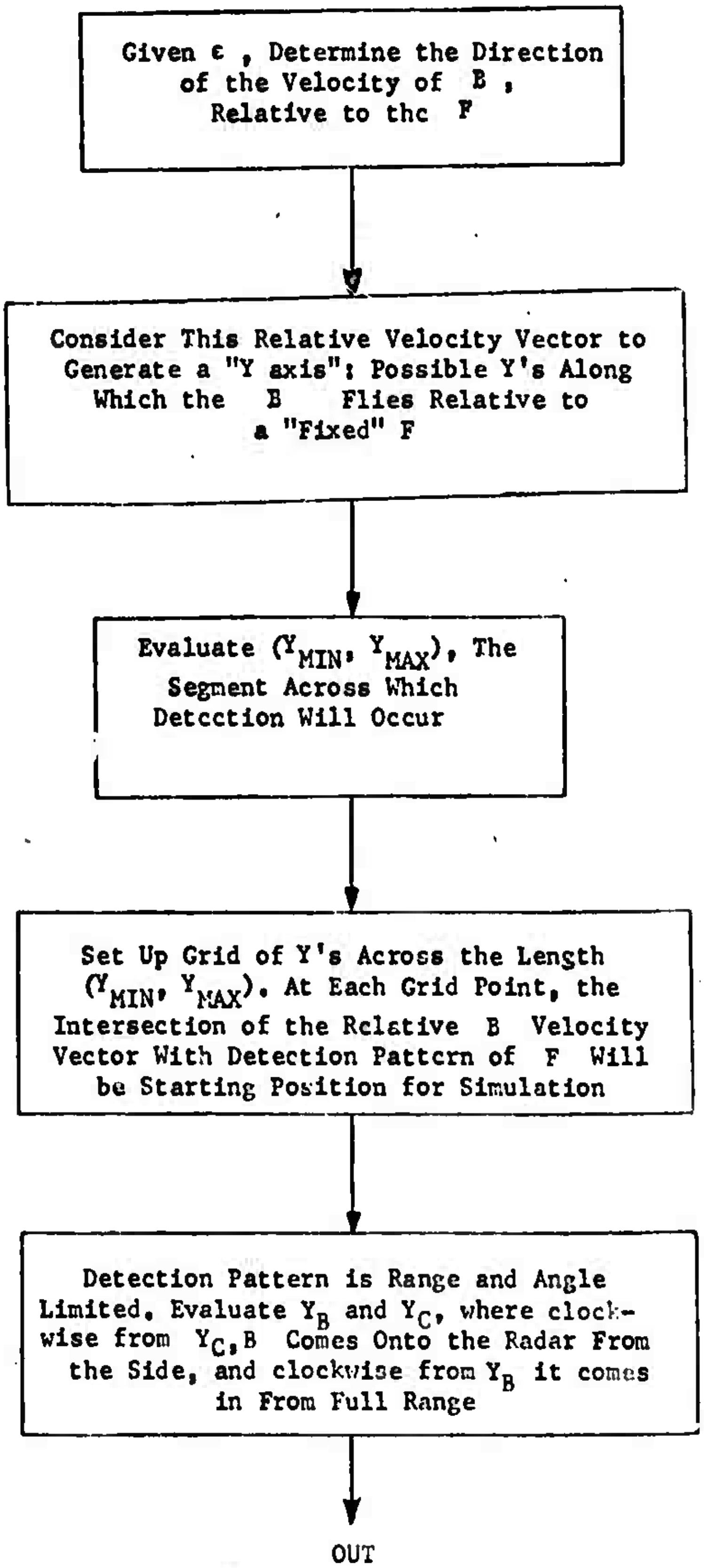


Figure 4.2-10 Grid for Given ε

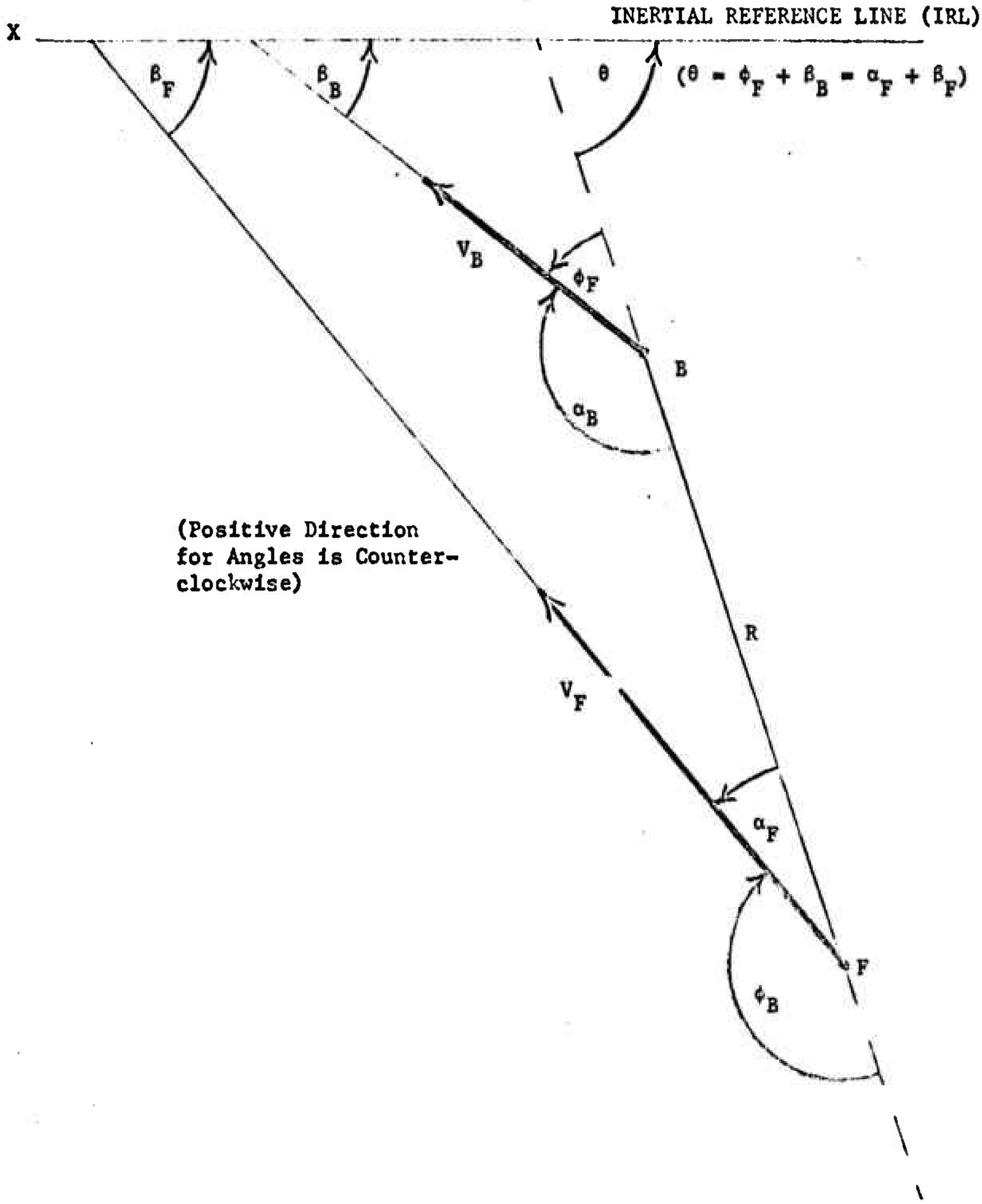


Figure 4.3-1 Defining Relative Positions

the negative of the target's velocity vector and the line of sight is referred to as the angle-off. These variables are subscripted B or F, as they relate to the bomber or the fighter. The angles are related by

$$| \alpha_B | = \pi - | \phi_F | \qquad (4.3\text{-}1)$$

$$| \alpha_F | = \pi - | \phi_B | \qquad (4.3\text{-}2)$$

The angles α and ϕ, along with the range R and speeds V_B and V_F are all that would be needed to describe the relative geometry if the bomber flew straight. In that case, the line of sight would turn precisely with ϕ_F. With both aircraft maneuvering, however, the turning rate of the line of sight, $\dot{\theta}$, changes with $\dot{\phi}_F + \dot{\beta}_B = \dot{\phi}_B + \dot{\beta}_F$, where $\dot{\beta}_B$ is the turning rate of the bomber. To keep track of $\dot{\beta}$ an inertial reference line is needed, the x-axis being as good as any; the direction of this line is arbitrary, as the values β and θ are not essential for the tactics of the aircraft, only $\dot{\beta}$ and $\dot{\theta}$. Knowledge of these various parameters is dependent on sensor information. In fact, the sensors are assumed to be perfect.

Although both aircraft will change their velocities V and turning rate $\dot{\beta}$ as they fly, the instantaneous effect of their V and $\dot{\beta}$ on the position variables α, ϕ, R, θ can be evaluated. The procedure is given in the next section.

4.3.1 Equations of Motion

Let a bomber B and a fighter F fly with speeds V_B and V_F respectively. Define the inner line of sight (LOS) as that portion of the LOS that is between the aircraft and the outer LOS as the remaining portions.

Define α_i to be that angle measured from the inner LOS to the heading of aircraft i and the angle ϕ_i as that angle measured from the outer LOS to the heading of aircraft j, i, j = F or B, i and j distinct. Also define β_i as tha angle between the heading of aircraft i and the inertial reference lina (IRL) measured as shown in Figure 4.3-1. All angles ara positive if measured in a counterclockwisa direction and negative otherwise. Thus, in the Figure α_F, ϕ_F, β_B, β_F, θ are all positive while α_B and ϕ_B are negative.

Let a dot $\cdot$ indicate the derivative with respect to tima. Then $\dot{R}$ is the rate of change of range between the two aircraft, $\dot{\theta}$ the turning rate of the LOS, $\dot{\alpha}_F$ tha turning rate of the tracking angle of the fighter and $\dot{\phi}_F$ the turning rate of tha fighter's angle off the bomber. With these definitions in mind, the four equations of relative motion are

$$\dot{R} = V_B \cos \phi_F - V_F \cos \alpha_F \qquad (4.3.1\text{-}1)$$

$$R\dot{\theta} = V_F \sin \alpha_F - V_B \sin \phi_F \qquad (4.3.1\text{-}2)$$

$$\dot{\alpha}_F = \dot{\theta} - \dot{\beta}_F \qquad (4.3.1\text{-}3)$$

$$\dot{\phi}_F = \dot{\theta} - \dot{\beta}_B \qquad (4.3.1\text{-}4)$$

The equations are derived in Appendix B.

These equations together with (4.3-1) and (4.3-2) are numerically integrated in the simulation by assuming that for one time pulse Δt, the V and $\dot{\beta}$ are held constant. Letting $\Delta t = dt$, we have:

$$R(t + \Delta t) = R(t) + \dot{R}\Delta t \text{ ; similarly for } \alpha_i, \beta_i, \phi_i .$$

The functional notation $R(t + \Delta t)$ means the value of R at the time $t + \Delta t$ in the simulation. Similar notation for other variables will subsequently be used. For the tactical and weapon firing decisions to be made by each pilot, the relative coordinates R, α_i, ϕ_i are used rather than the inertial coordinates x_i, y_i. Nevertheless, for an analyst to review an engagement, a plot of the relative coordinates (R, ϕ) may not be so enlightening as an inertial coordinate (x, y) plot. As $\dot{\beta}_B$ is known Figure 4.3-2 gives the values of $\dot{x}_B$ and $\dot{y}_B$ for the bomber immediately:

$$\begin{aligned} \dot{x}_B &= - V_B \cos \beta_B \\ \dot{y}_B &= V_B \sin \beta_B \end{aligned} \qquad (4.3.1\text{-}5)$$

As before, $x_B(t + \Delta t) = \dot{x}_B(t) + x_B \Delta t$. Similarly for $y_B(t + \Delta t)$. Thus the new (x_B , y_B) position of the bomber is obtained.

To find the new position of the fighter, equations (4.3.1-5) subscripted with F instead of B are valid. However, in order that the relative geometry be as exact as possible, the fighter's coordinates are derived from the bomber's (see Figure 4.3-3):(1)

$$\begin{aligned} x_F &= x_B + R \cos (\phi_F + \beta_B) \\ y_F &= y_B - R \sin (\phi_F + \beta_B) \end{aligned} \qquad (4.3.1\text{-}6)$$

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(1) Figure 4.3-2 and Figure 4.3-3 are quite different conceptually. Note the meaning of the hypotenuse in each case.

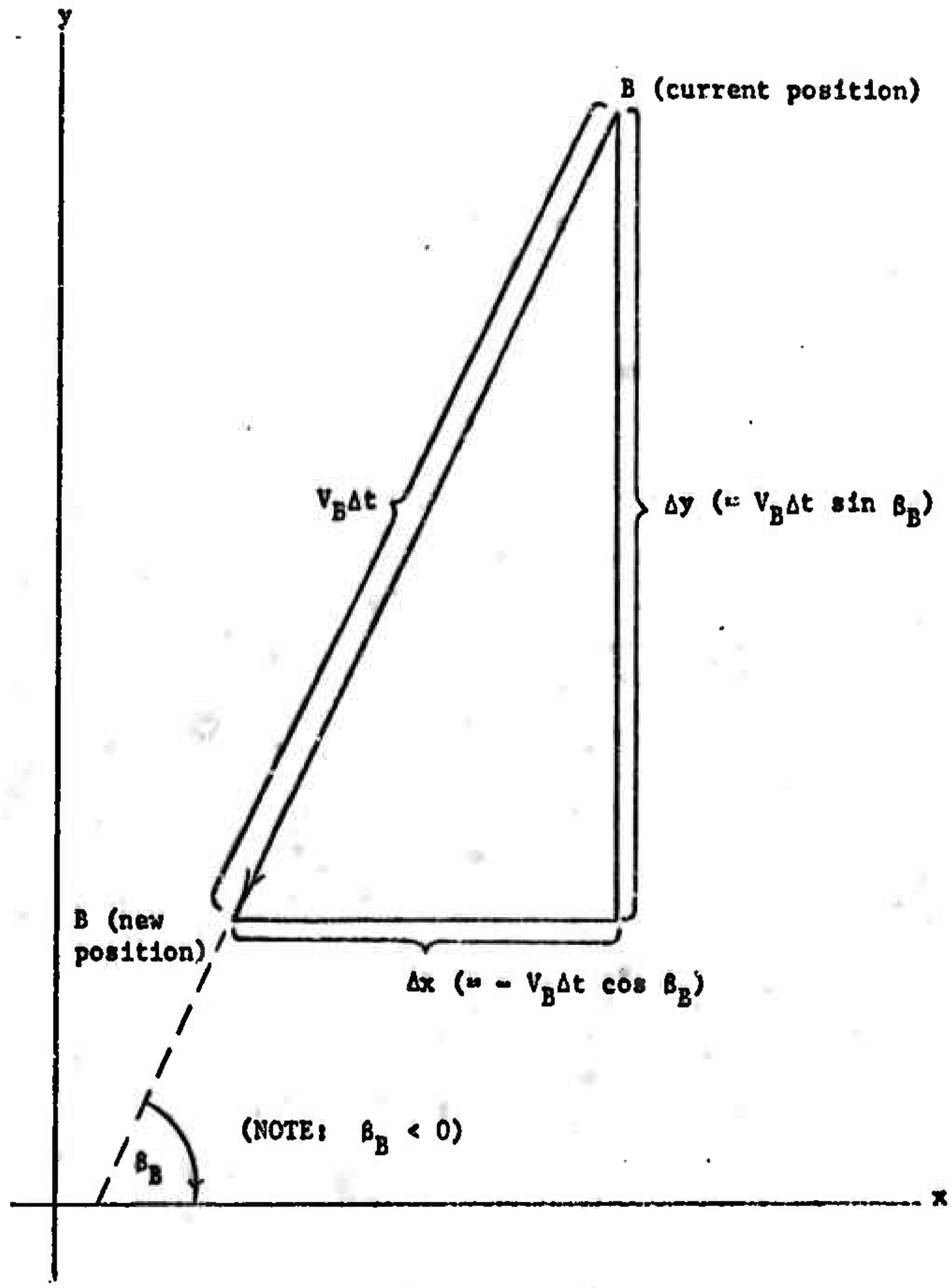

Figure 4.3-2 Establishing Inertial Position of Bomber

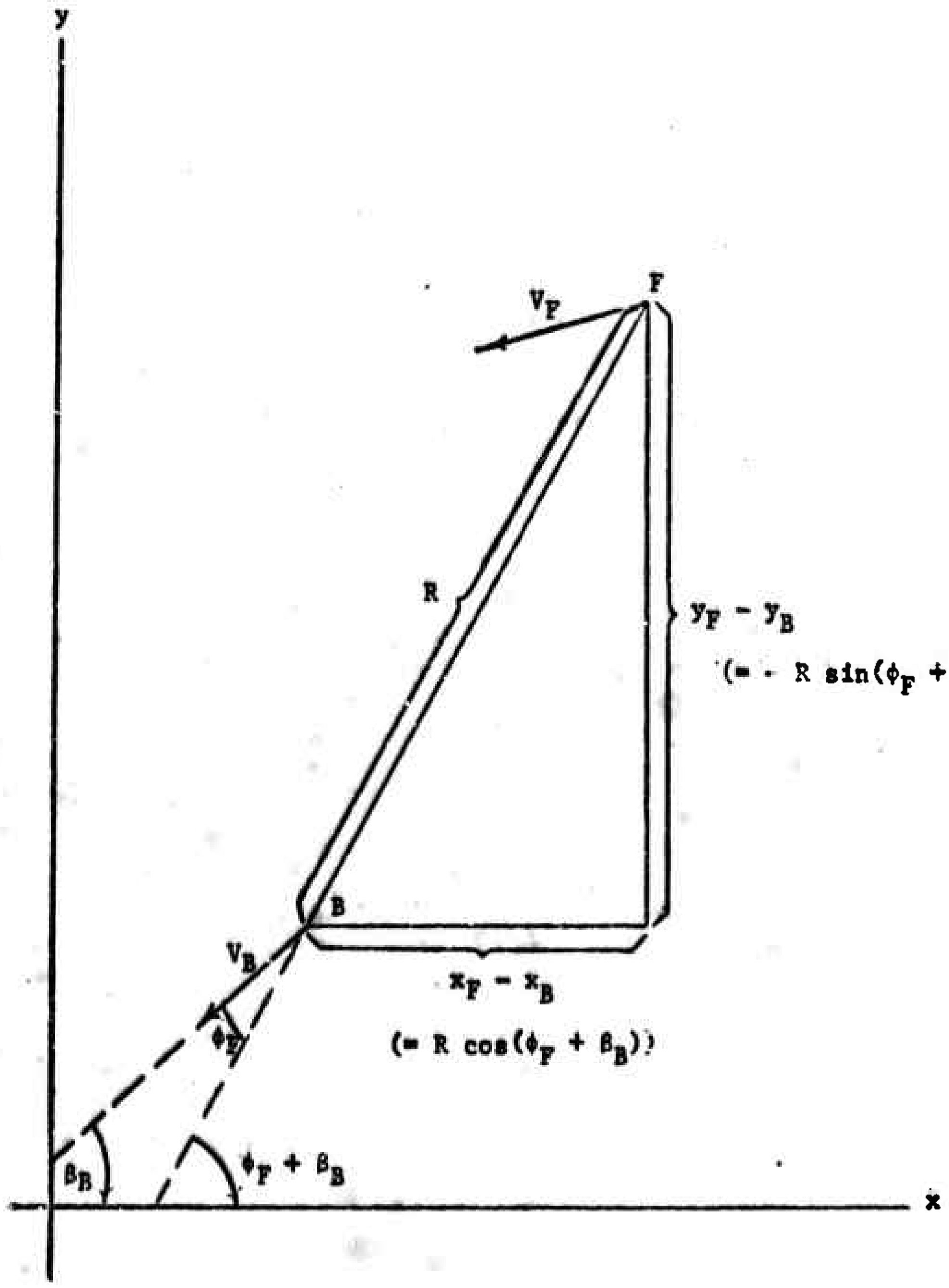

Figure 4.3-3 Establishing Inertial Position of Fighter

It is then necessary to increment the velocity. Let a_i be the acceleration of aircraft i at time t. Then

$$V_i(t + \Delta t) = V_i(t) + a_i \Delta t \quad .$$

This completes the usual procedure for mathematically evaluating the motion of the aircraft. An alternative approach is sometimes used and is described below.

4.3.2 <u>Equations Close In</u>

If the two aircraft cross each other's path at close range, the line of sight turns rapidly. The usual linear integration of the equations of motion form a bad approximation in this case; although for an instant the line of sight may turn at, say 1440° per second, this does not hold true for so long as the 1/4 second discrete jump of the simulation, but only for the instant of time the aircraft cross. The usual approximation would alter the line of sight by

$$\theta(t + \Delta t) = \theta(t) + \dot{\theta}\Delta t = \theta(t) + 360° \ .$$

The aircraft in the model would perform a loop-the-loop. To avoid this problem the (x, y) coordinates are updated directly without using equations (4.3.1-1) through (4.3.1-4). Knowing the bomber's turning rate $\dot{\beta}$, an updated β is found, and x and y are found as before (equation 4.3.1-5). Now the relative variables are found from (see Figure 4.3-3)

$$R = \sqrt{(x_B - x_F)^2 + (y_F - y_B)^2} \quad ,$$

$$\tan(\alpha_F + \beta_F) = \tan(\phi_F + \beta_B) = \frac{y_B - y_F}{x_F - x_B}$$

This procedure is not used universally, as it is more important to be faithful to the relative geometry of the aircraft than the (x, y) coordinate system.

4.4 Weapon Firing

The procedure for the simulation then, is to select the acceleration (a) and turning rate ($\dot{\beta}$) parameters (which are, to an extent, under the control of the pilot) and run these values through the equations of motion, and thus arrive at a new position. In order to consider the pilot's capability to set these a and $\dot{\beta}$ values, and the related question of what the tactical doctrine is when a choice is available, it is necessary to see what are desirable positions in combat. These are determined in part by the weapon loading of the aircraft.

A successful kill from the firing of any weapon is dependent on many conditions. Among them:

1. Aircraft tracking ability
2. Target position and maneuver
3. Missile lock-on ability
4. Fusing-aiming error
5. Reliability of Missile
6. Component Structure of Target

The first three points are considered in the EM directly. Except for tail guns, a weapon will not be fired if the aircraft does not track the target.[1] The tracking radar pattern of the aircraft is assumed to be a

---

(1) Throughout the discussion, the terms pursuer, target will be used to refer to either aircraft, as each considers the other the target. The subscripts P and T on variables will mean pursuer and target, respectively.

cone just as is the detection radar, with possibly a different range and angular coverage. So long as the target is inside this pattern it can be tracked.

4.4.1 Launch Envelopes

A missile must be able to catch the target without exceeding its maximum g capability, and without running out of power. If the target holds to a fixed turn at a constant speed, then it is possible to construct range-angle off contours describing the geometric limits for a successful launch. Referred to as envelopes, these contours are inputs to the model. An example of these weapon envelopes appears in Figure 4.4-1. The target moves at speed V_T, at a certain g loading, and is turning to the left. If the target turned to the left, the envelope would be replaced by its mirror image. Typically, as the target speed and g loading increases, the weapon envelope understandably shrinks. A pursuer inside the envelope, but outside the inner barrier, can fire and expect the missile to maintain course up to the target. The inner barrier exists usually because of g missile limitations; The missile must immediately at launch pull too hard. The irregular shape of the envelope results because different target aspects will require different maximum g loadings somewhere along the path of the missile. Also, the signal return from the target may be considered in forming the envelope. An IR controlled missile may, for example, not receive a signal off the nose of a subsonic target.

Missile lock-on depends not only on range and angle-off, but on α, the tracking angle. All weapons except tail guns are assumed fired off the nose of the pursuer. Within an angle α_{MIS} off the nose, the missile may lock-on. Thus, the aircraft need not point directly at the target, only within α_{MIS}.

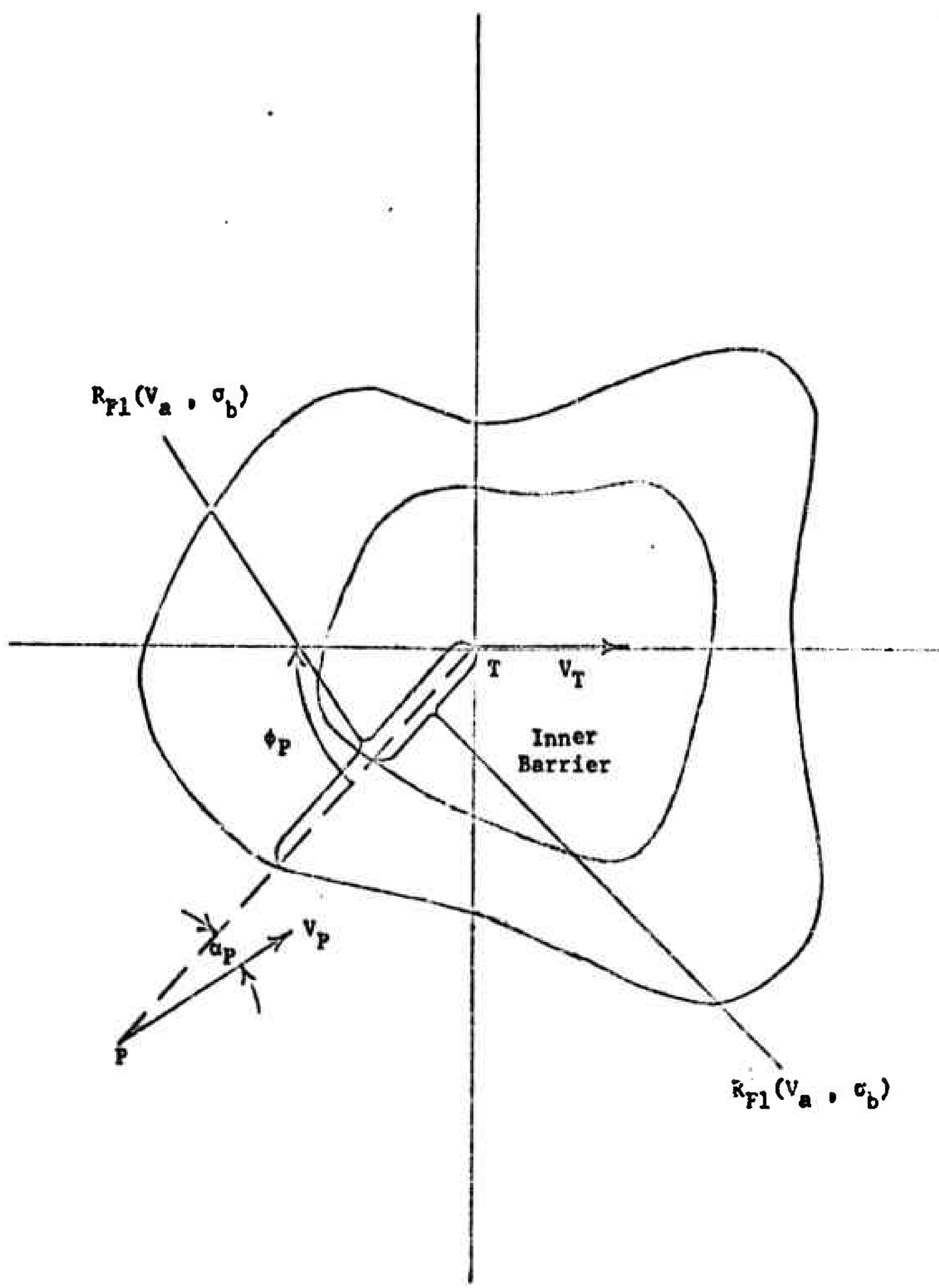

Figure 4.4-1 Weapon Firing Envelope

4.4.2 Lethality

Points 4, 5, and 6 are not treated directly in the model. Detailed consideration of these factors would place too much emphasis on the weaponry. In the DPM, however, they are treated in the sense of a degradation factor on firings. Based on assumptions about the fusing and aiming errors, a probability of coming sufficiently close to the target and detonating at the right time can be determined. In addition, a probability may be associated with the reliability. To take care of the component structure, the probability of hitting a vulnerable part of the target may be estimated. Combining these probabilities gives the probability of kill by the weapon, conditional, of course, on being fired within the geometric limitations described above. This probability of kill is an input to the DPM of ATAC-2.

A tacit assumption above is that regardless of where inside the launch envelope the weapon is fired, the fusing and aiming errors and the likelihood of hitting a vulnerable spot on the target, remain the same. This is not true in general. Aiming will be better up close, and certain aspects of the target present greater vulnerable area. However, the kill probability (PK) is assumed not to vary enormously within the envelope region, particularly for missiles. But suppose a weapon's PK does change sufficiently with position as to warrant special evaluation. The weapon then may be split up into, say, two weapons each with a distinct PK , and with mutually exclusive launch envelopes (see Figure 4.4-2). Missile number 1 has an inner barrier not defined by g limitations, but by the fact that missile number 2 (the same missile) may be fired in that region with a higher PK .

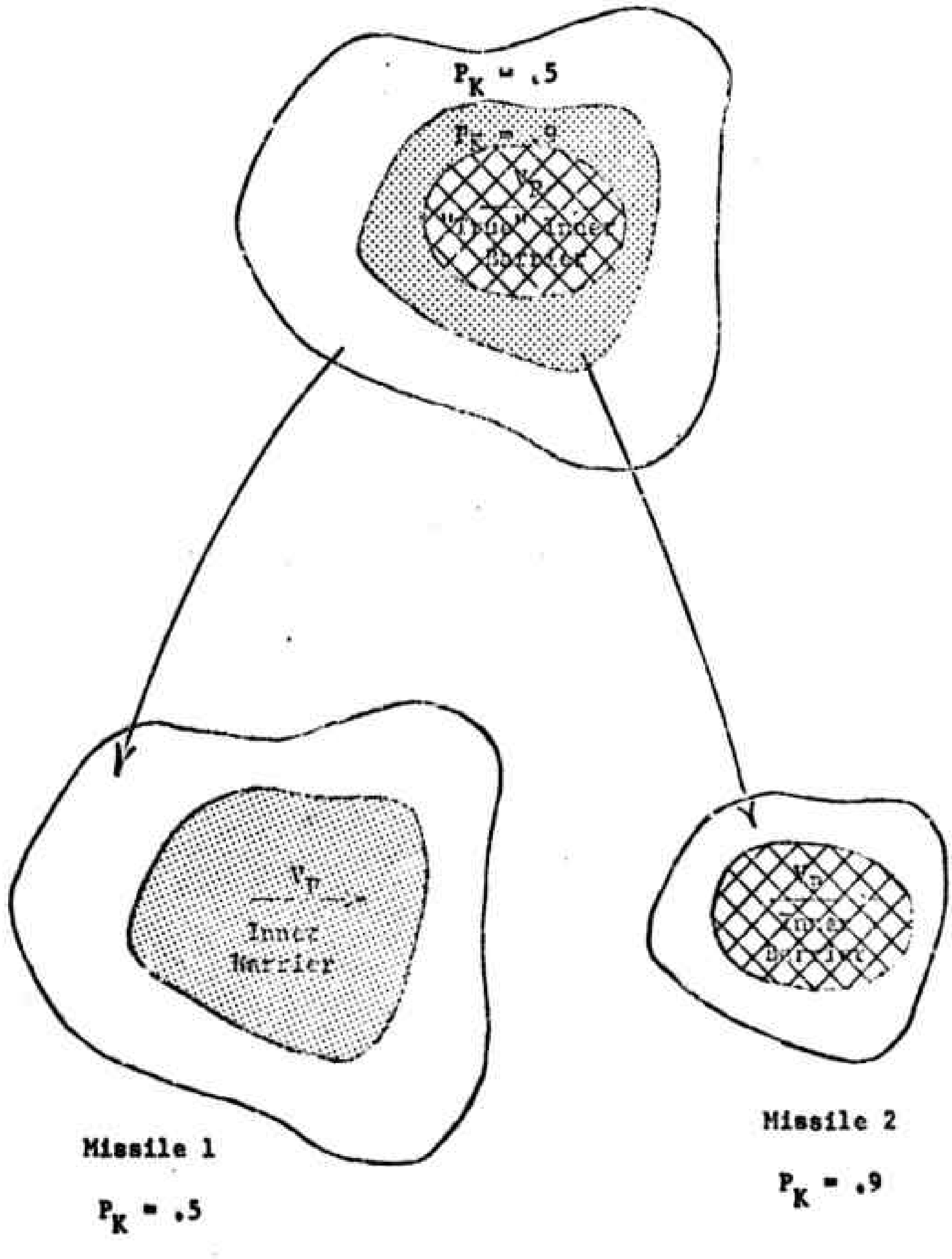

Figure 4.4-2 Handling Missiles with Varying Lethality

All of the above geometry describes when a missile can be fired. The model assumes a skillful pilot who will in fact try to fire whenever the geometric conditions are met. Further, the pilot will never fire otherwise. A pilot in reality will occasionally waste ammunition with a poor shot, but such concerns do not add to the evaluation of aircraft, weapons, and tactics for which ATAC-2 exists. (The input kill probabilities may include a degradation factor for this.) Nevertheless, the pilot is subject to some restrictions which are contained in tha model, discussed next.

4.4.3 IFF

First, IFF must be establishad. Within a certain range-angle pattern similar to the detection and tracking radar and optical sighting, an aircraft could be observed to be friendly (or return a signal so announcing). Obtaining IFF may well be required optically although radar equipment is available. Until the target is inside tha IFF pattern, no weapon may be fired. Once inside, though, tha pilot can fire under suitabla conditions after the enemy is no longer in the IFF pattarn -- presumably if the pilot can track the target he can tell tha target is tha previously identified foe.

4.4.4 Oxygen Debt

Another pilot restriction, besides IFF, is hia physical condition [Ref. 3]. Pulling too heavy a g loading wears a pilot down. Blood leaves the brain, or is not rasupplied. This causes oxygen depletion in the brain, which makes it harder for a pilot to make good decisions. An expariencad pilot can control this oxygen drain, by muscle constriction in hia abdominal region but he will eventually tira of this. Fatigue sets in. This phenomenon of incurring an oxygen debt is incorporated as follows:

Above a loading of 4 g's the oxygen debt is incurred at a rate which is a function of the g loading

$$dO = f_1(g)\ dt \quad ,$$

where O is the oxygen debt in the blood. As oxygen debt is incurred, the firing of weapons may be retarded. Below a loading of 1.5 g's the oxygen debt is reduced at a constant rate

$$dO = -f_2\ dt \quad .$$

This oxygen debt rate is integrated over time. When the oxygen debt reaches a certain leval, the pilot is said to be sick or tired. Depending on inputs, the pilot may be required to fly at less than 1.5 g's until his oxygen debt is cancelled.

4.4.5 Firing Rate

When all conditions for firing are met, tha supply of weapons may be fired until exhauated, so long as the conditions remain satisfactory. Weapons of a given type are fired at a fixed rate, unless retarded by tha oxygen debt, described in Section 4.4.4.

4.4.6 Tail Guns

If an aircraft is equipped with tail guns, these must be treated distinctly from other weapons. All other weapons are fired within a certain angle off the nose. The tail gun has associated with it an angle α_g , similar to α_{MIS} of ther weapona, but the meaning of this angla is the maximum angle off the tail within which the guns may be fired. Mathematically, tail gun firings must satisfy

$$|\alpha_i| \leq \pi - \alpha_g(i)$$

Ordinarily, the tracking and detection radar cannot operate on a target in the rear, and thus tail guns in the model may be fired without the need of the usual requisite information on the target. Nevertheless, tail guns firings are governed by a weapon envelope like other weapons. The model essentially treats the tail guns as if a separate individual, isolated from the pilot, controlled them.

4.5 Attacker's Course

The weapon loading dictates some of the navigation policy. First of all, in attempting to convert on the opponent, coming in from behind is manifestly desirable. Most weapons are fired easier from behind, so that the aircraft can aim well and fire while the opponent cannot. Further, a position from the rear can be maintained longer, resulting in more shots fired. Finally, if the target cannot see out the rear, the aircraft may achieve a surprise attack.

4.5.1 DEL Pursuit Course

Figure 4.5-1 describes the method of coming from behind. The target is fixed. The policy is defined by the angle-off, ϕ. When $\phi = 0$ the pursuer is directly behind the target, which will be an objective. When ϕ is large (> 90°) the opponent is headed towards the aircraft. To counter this the pursuer points behind the target. This means α, the tracking angle of the pursuer is non-zero ($\alpha = 0$ is pointing at the target). Such an α is called a lag angle as opposed to lead (pointing in front). As the angle-off decreases and the attacker arrives at firing position, it must point

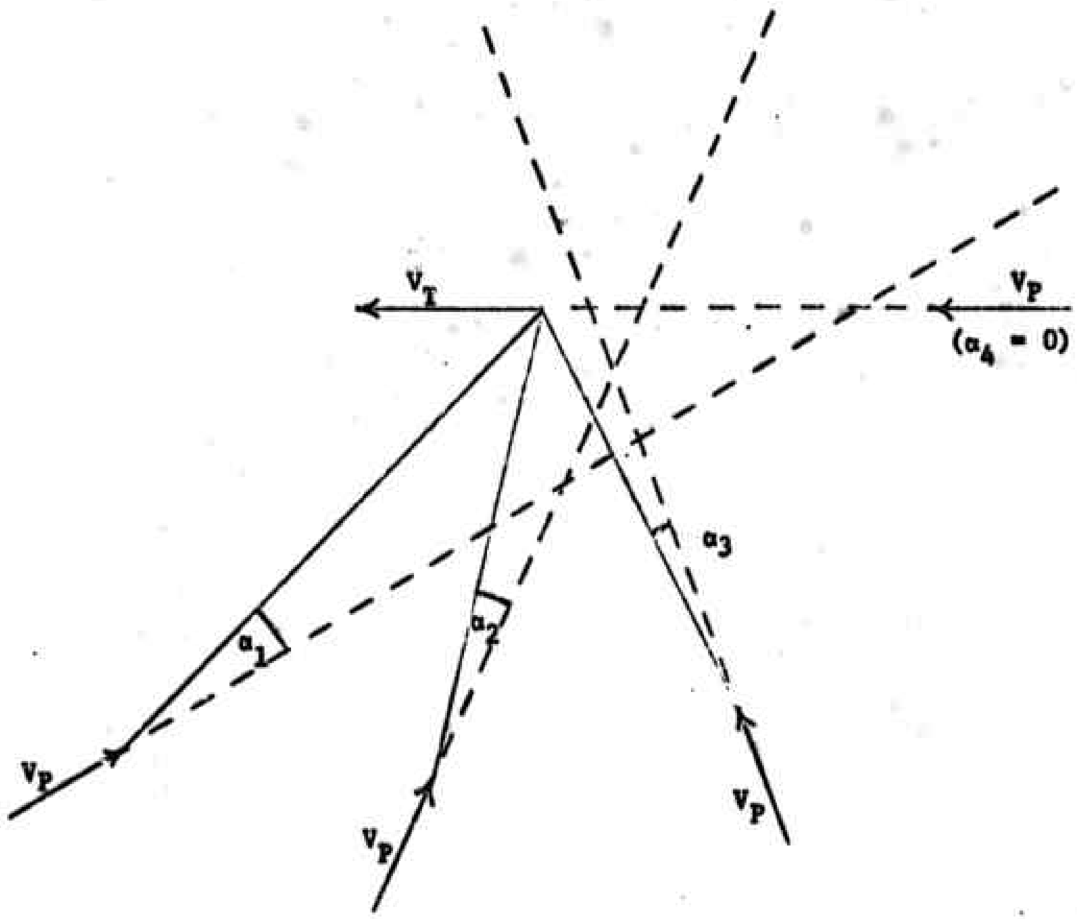

Figure 4.5-1 Decrease in Lag as Pursuer Swings Behind Target

close enough at the target to fire (the α_{MIS} condition of the weapons, described above in Section 4.4.1). Thus, the lag angle α should reduce as the angle-off ϕ reduces. In Figure 4.5-1, $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4$.

The equations are given below to describe this decreasing lag pursuit course, or DEL pursuit course for short.

$$
\begin{aligned}
|\alpha| &= K(\phi - \phi^*) + \lambda \quad , \text{ if } \phi \geq \phi^* \quad , \\
|\alpha| &= \lambda \qquad\qquad\qquad , \text{ if } \phi < \phi^* \quad ,
\end{aligned}
\qquad (4.5\text{-}1)
$$

where K is a scaling factor. A full discussion of the DEL pursuit course is given in Appendix C.

The equations say that as the pursuer swings behind the target, the lag angle α should decrease, until the angle-off ϕ is less than ϕ^* . The lag angle remains constant (λ) from then on in. If $\lambda = 0$, then within ϕ^* the course is pure pursuit. If $\lambda \neq 0$, the course within ϕ^* is called constant lag pursuit. The term "pursuit" in subsequent discussion will refer to the general course described by equation (4.5-1).

The angle ϕ^* can be interpreted as a cone at which maximum g's will be pulled on a pure pursuit course by a constant speed attacker. The idea here is that if the attacker can cross this barrier, a pure pursuit is possible from there on in. This rests on the assumption that the target proceeds to fly at constant speed and direction. Actually the target will probably maneuver (both in reality and in the model) and the pursuer may be unable to maintain pursuit. Section C.4 shows that ϕ^* may be described by

$$\cos \phi^* = \frac{V_P}{2V_T} \quad .$$

If the pursuer flies at least twice as fast as the target, then $\psi^* = 0°$. If pure pursuit is intended eventually ($\lambda = 0$ in equation (4.5-1)) and if the pursuer first attacks the target from in front, the pursuer will point behind the target until it is directly behind, and only then will fly pure pursuit. If $V_P < 2 V_T$, then the DEL course degenerates to pure pursuit before the pursuer swings entirely around behind the target. If when the attack begins, the pursuer is sufficiently behind the target to want to be on pure pursuit, then, of course, it adjusts to pure pursuit as fast as it can.

Both aircraft will normally attempt to perform the same sort of maneuver. The aircraft with the best performance characteristics will presumably be the one to get on the tail of the other.

4.5.2 Preferred Firing Positions (R*) and Closing Speed Doctrine

When an aircraft is on pursuit, the turning rate $\dot{\beta}$ is determined by the equations for $\dot{\alpha}$ (Equation 4.3.1-3)). Speed, however, can vary also. If directly behind the target's tail, but far to the rear, then clearly the pursuer must go faster than the target; otherwise it never catches up. On the other hand, it must not go too fast or it may find itself so close to the target that even if it decelerates as hard as possible it must pull away, to avoid colliding or passing the target. Ideally, as it closes in, it will shoot its longer range air-to-air missiles if it has any. This firing will occur at a certain desired range (initially set as R*) if the pursuer has maintained the surprise element. If the target survives these weapons, the pursuer will close to an ideal range (resetting R*) for its shortest range weapons (presumably guns). At that point it should move at nearly the target speed for maximum length contact. Since the pursuer should move in as quickly as possible in order to allow minimum time for target maneuver, the doctrine says that while on pursuit an aircraft continuously speeds up until the point where maximum slow down will place the pursuer just at R* when flying at the target speed.

4.6 Turning Rate ($\dot{\beta}$) and Acceleration (a) Tactics

In Section 4.6 the subscript "i" to represent the aircraft, is suppressed.

4.6.1 Doctrine's Desired $\dot{\beta}$ and a

The doctrine of desirable turning rate and and acceleration is now stated, presuming the aircraft can see the target. Assuming unlimited turning capability we formulate, first, the turning rate, and second, at what point to switch from acceleration to deceleration. The restriction of limited turning rate is later applied (in 4.6.2.2).

4.6.1.1 Turning Rate

It is evident from equation (4.3.1-3) that by turning exactly as fast as the line of sight ($\dot{\beta} = \dot{\theta}$) the aircraft holds its tracking angle α constant. If, on the other hand, a new α (= η) is desired to maintain the DEL pursuit course during the pulse, then the necessary turning rate is

$$\dot{\beta} = \dot{\theta} - \dot{\alpha} \simeq \dot{\theta} - \frac{\Delta\alpha}{\Delta t} = \dot{\theta} - \frac{\eta - \alpha}{\Delta t} \quad .$$

After exercising the equations of motion with this $\dot{\beta}$ the tracking angle equals η . The lateral g loading g_1 associated with this $\dot{\beta}$ is

$$g_1 = \frac{V\dot{\beta}}{32.2} \quad .$$

from the standard equations for centrifugal acceleration. The total g loading, g_2, including that due to the downward gravitational force is

$$g_2(\dot{\beta}) = \left[1 + \left(\frac{V}{32.2}\right)^2\right]^{1/2} . \qquad (4.6\text{-}1)$$

The inverse function, evaluating a turning rate for a given g loading (g_2) is often used:

$$\dot{\beta}(g_2) = \frac{32.2}{V} \quad (g_2^2 - 1)^{1/2} . \qquad (4.6\text{-}2)$$

The last two equations show that for a fixed velocity, a turning rate specifies the g's pulled, and vice versa. The model concerns itself primarily with tactics which capitalize on the turning rate rather than g's. This is not arbitrary. It gives rise to better tactics.

Appendix D discusses this point.

4.6.1.2 Criterion for Acceleration

The basic acceleration doctrine is to accelerate as much as possible until it is imperative to slow down. To define the point where deceleration is necessary, suppose it will eventually be desirable to fly at a speed V'. Let a pursuer travel at speed V_P and continuously decelerate at a constant rate a_{DEC} (a negative quantity) to V'. Let the target travel at a constant speed V_T. Define a function $S(V')$ which is an estimate of the distance the pursuer closes in on the target by the time the pursuer's speed equals V' — an estimate only, since it assumes linear flight by both aircraft. The time it takes to decelerate is

$$\frac{V_P - V'}{- a_{DEC}} \quad .$$

The average rate of change is

$$\frac{V_P + V'}{2} - V_T \cos \phi \quad ,$$

the average speed of the pursuer less the projection of the target velocity on the line of sight (see Figure 4.6-1). Thus,

$$S(V') = \frac{V_P - V'}{- a_{DEC}} \left(\frac{V_P + V'}{2} - V_T \cos \phi \right) = \frac{(V_P - V')(V_P + V' - 2V_T \cos \phi)}{- 2\, a_{DEC}}$$

(4.6-3)

This function guides the pursuer in determining whether it must slow down.

4.6.2 Limitations on Maneuvers

Acceleration is not unlimited. At any g loading there is a minimum speed to avoid stalling and a maximum speed either due to power limitations or overheating. Further, at any g loading and speed, the power (throttle) setting limits the acceleration of the aircraft. The turning rate is also limited. Moreover, acceleration and turning rate are interrelated.

4.6.2.1 Specific Power, P_S

The limitations are described in the Specific Power function for the given power setting. At high g and high speed, the function may be negative, meaning the aircraft can only slow down. If it is to go faster it must reduce its turning rate. The Specific Power Function is of the form:

$$P_S = P_S(V, g_Z) \quad .$$

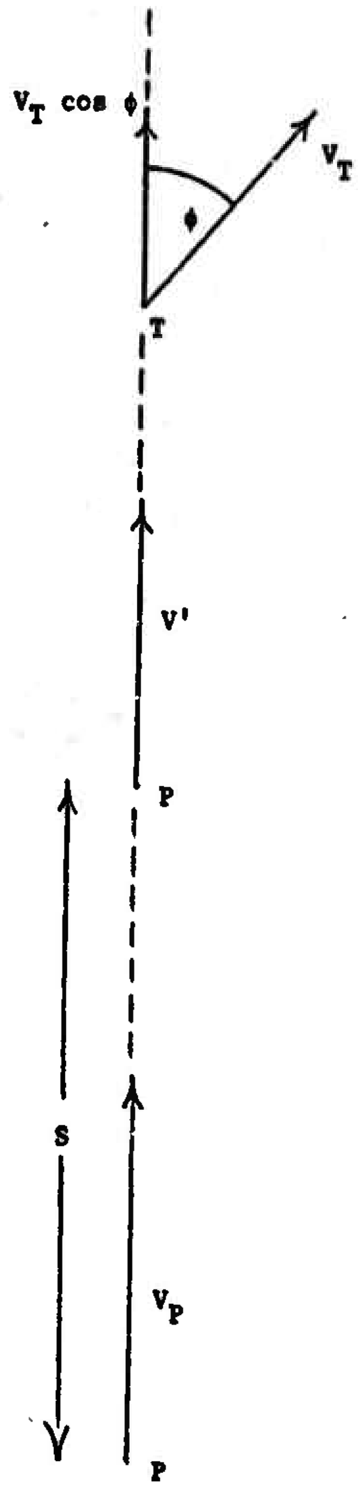

Figure 4.6-1 Geometric Description of S(V')

See Figura 4.6-2 for an example (although artificial). If $P_s < V$, it is equal to the maximum rata of climb at velocity V, g loading g_2. Appendix E discusses in detail this function. Multiplying by $32.2/V$ gives, for acceleration in a horizontal plane:

$$a = \frac{32.2}{V} P_s(V, g_2) \quad .$$

The aircraft can accelerate up to this amount. It can control its speed by increasing drag, allowing it to slow down as much as by a constant rate a_{DEC}; if a is smaller (more negativa) than a_{DEC}, however, it must slow down by a. The simplifying assumption of the model is that the throttle setting is not changed during the engagement; this one function $P_s(V, g_2)$ describes the entire turning and acceleration capability of tha aircraft.

4.6.2.2 General Rule for Handling P_s Limitations

The restriction on acceleration and turning rate may prevent the aircraft from speeding up. For example, if P_s is negative tha aircraft cannot turn consistent with pursuit and simultaneously speed up. To see what an aircraft might do, consider that it is generally undesirabla to fly at a speed that is not maintainable, which could happen when excessive g's are pulled. Define $g(V)$, such that

$$P_s(V, g(V)) = 0 \quad .$$

Then $\dot{\beta}(g(V))$ as given by equation (4.6-2), is the maximum turning rata at which the speed V can be maintained. For example, from Figure 4.6-2, at 1,000 ft/sec, $g(V) \approx 6.2$. With certain exceptions noted later, the doctrine is to turn at a rata no more than $\dot{\beta}(g(V))$. When on pursuit, this means that the aircraft will occasionally stray from the doctrine DEL.

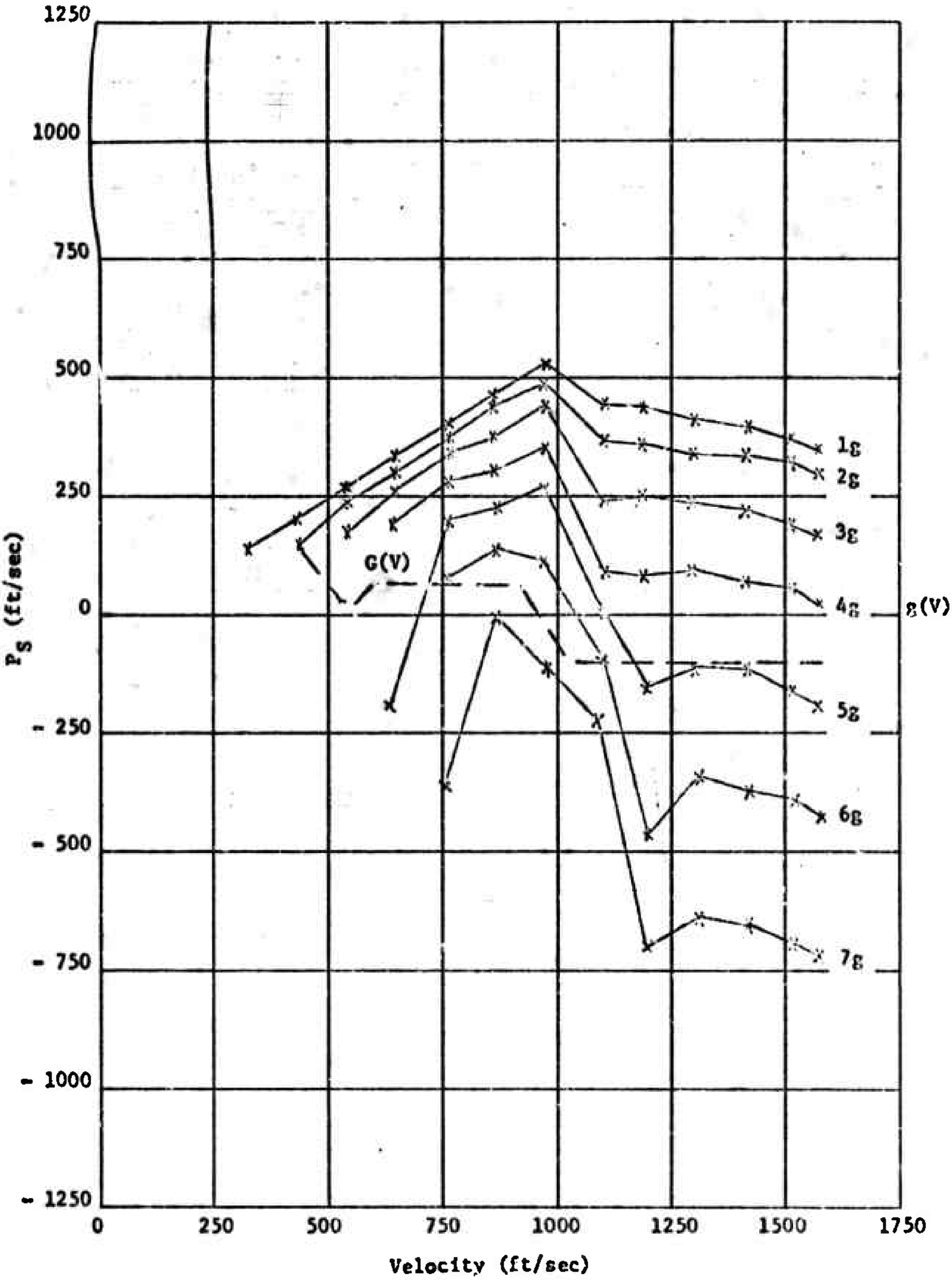

Figure 4.6-2 Typical Specific Power Function

pursuit course rather than be forced to lose speed, when speed loss is also undesirable.

4.6.2.3 Other Limitations

Sometimes $g(V)$ is unattainable. There is an upper limit to the pilot's ability to withstand the g's. In addition, when the pilot is deemed "sick" from too high a g loading, he is required to fly fairly straight (< 1.5g) for some time to recuperate. Further, there are structural and aerodynamic limitations on tha aircraft, and too many g's can pull the aircraft apart. These structural limitations on g's ara a function of velocity, described by $G(V)$. This function is described by the dotted line in Figure 4.6-2. It is an implicit description, for to find $G(V)$ for a given V one must locate where the function lies relative to the various g levels. If $G(V)$ is less than $g(V)$, of course, the aircraft is limited by $G(V)$. For example, from Figure 4.6-2, the aircraft cannot turn at the rate $\dot{\beta}(g(V))$ until the speed is greater than 950 ft/sec. This does not mean it necessarily must accelerate, since it can increase drag; only that, say, at 750 ft/sec it cannot pull more than 6 g's.

On the other hand, the aircraft may be unable to maintain speed aven while flying straight and level (presumably this state of affairs came about from some other power setting or a change of altitude). The model requires that the aircraft never begin an encounter from such high speeds, to avoid this problem.

As noted from equation (4.6-2) for a given velocity, pulling a certain number of g's implies a certain turning rate, $\dot{\beta}$. The tactics relate directly to $\dot{\beta}$, and thus the specific power is used in the model as the function $P(V, \dot{\beta})$ rather than the function $P(V, g)$.

4.7 Details of Tactics

In Section 4.7, the subscript "1" to represent the aircraft, is suppressed.

4.7.1 Pursuit Doctrine

Assume now the pursuer has the target on its detection radar. This is considered sufficient for the pursuer to know the entire geometry of the situation. Suppose further that the necessary turning rate $\dot{\beta}$ is less than $\dot{\beta}(g(V))$, i.e., the aircraft need not slow down in order to turn at the required rate. (The single exception (Case 1) will be noted below.) We now construct the details of the pursuit doctrine.

To determine the appropriate acceleration, a multitude of special cases arise. First, let the pursuer attack from ahead of the target ($\phi > 90°$). Then eventually it will swing behind the target. Around crossover turning will be tight, so that the desired velocity is V^*, the velocity at which it can turn the best ($\dot{\beta}$ is maximal for a sustained period). For most aircraft such an optimal velocity exists. Then, if the pursuer manages to turn behind the target, the desired speed will eventually be adjusted to the target's as the angle-off goes to zero.

If the pursuer is in front and flying faster than V^*, it may not yet be necessary to decelerate. The attacker still wants to come in as fast as possible and slow down only when necessary. The function $S(V')$, defined in equation (4.6-3), offers a criterion; if $S(V^*)$ is greater than the range, this means that the aircraft will be going faster than its best turning speed when close to the target (Case 6 of Figure 4.7-1) and thus

PRESENT STATE OF AIRCRAFT i (10)

Case # (1) (m State)	Policy	Inf. Available	Angle-Off	SPEED: Main Condition	SPEED: Other Conditions If Any	TURNING RATE (5) RELATIVE TO OPPONENT: Main Condition	TURNING RATE (5) RELATIVE TO OPPONENT: Other Conditions If Any	RANGE: Main Condition	RANGE: Other Conditions If Any
0	Attack	Unaware	—	—	—	—	—	—	—
1a		Active	$\|\phi\| \le \pi/2$	$V > V_o$	—	—	—	$R < R^*$	—
1b1					$V \ge V_o - a\Delta t$ (4)	—	—	$R \ge R^*$	$R<R^*+S(V_o)-\dot{R}\Delta t$
1b2					$V < V_o - a\Delta t$ (4)	—	—		
2a					$V > V^*$	—	—	$R \ge R^*+S(V_o)-\dot{R}\Delta t$	—
2b					$V \le V^*$	—	—		—
3a				$V \le V_o$	$V > V^*$	—	—		—
3b					$V \le V^*$	—	—		—
4a					$V \ge y$	$\dot{\beta}(g(y)) \ge \dot{\beta}_j$ (9)	—		—
4b					$V < y$		—		—
4c1					$\exists y$,(8) $V > V^*$	$\dot{\beta}(g(y)) < \dot{\beta}_j$	—		—
4c2					$\nexists y, V > V^*$	—	—		—
4d1					$\exists y, V \le V^*$	$\dot{\beta}(g(V)) < \dot{\beta}_j$	$\dot{\beta}(g(y)) < \dot{\beta}_j$		—
4d2					$\nexists y, V \le V^*$		—		—
4e1					$\exists y, V \le V^*$	$\dot{\beta}(g(V)) \ge \dot{\beta}_j$	$\dot{\beta}(g(y)) < \dot{\beta}_j$		—
4e2					$\nexists y, V \le V^*$		—		—

Figure 4.7-1 Tactical Cases (m States)

RESPONSE FOR AIRCRAFT 1

Case #(1) (m State)	Acceleration	Acceleration Bounds(2)	Turning Rate(3)	Turning Rate Bounds(2)	Comments
0	$a = 0$	——	$\dot{\beta} = 0$	——	Fly straight until aware of fighter
1a	$a = a_{DEC}$	$a \leq a(P_S)$ (7)	$\dot{\beta} = \dot{\theta} + \frac{\alpha - \eta}{\Delta t}$	$\lvert\dot{\beta}\rvert \leq \dot{\beta}(G(V))$	Slow down to avoid overshoot
1b1	$a = a_{DEC}$	$a \leq a(P_S)$			Slow down to avoid overshoot
1b2	$a = 0$	——			Do not slow down, to be sure R* is reached
2a	$a = a(P_S)$ (6)	——		$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))$	Not yet the time to slow down
2b		——		$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))\frac{V}{V^*}$	Not yet the time to slow down
3a		——		$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))$	Speed up to catch target
3b		——		$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))\frac{V}{V^*}$	Speed up to catch target
4a	$a = a_{DEC}$	$a \geq \frac{V - V}{\Delta t}$		$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))$	Slow down to steady state velocity
4b	$a = a(P_S)$	$a \leq \frac{V - V}{\Delta t}$			Speed up to steady state velocity
4c1	$a = a_{DEC}$	$a \geq \frac{V^* - V}{\Delta t}$			Steady state impossible - slow down to V*
4c2					Steady state impossible - slow down to V*
4d1	$a = a(P_S)$	$a \leq \frac{V_1 - V}{\Delta t}$			Steady state impossible - speed up to target speed
4d2					Steady state impossible - speed up to target speed
4e1	$a = 0$	——			Steady state impossible - do not speed up, since target can turn better
4e2		——			Steady state impossible - do not speed up, since target can turn better

Figure 4.7-1 Tactical Cases (m States) (Cont.)

PRESENT STATE OF AIRCRAFT i (10)

Case # (1) (m State)	Policy	Inf. Available	Angle-Off	SPEED: Main Condition	SPEED: Other Conditions If Any	TURNING RATE (5) RELATIVE TO OPPONENT: Main Condition	TURNING RATE (5) RELATIVE TO OPPONENT: Other Conditions If Any	RANGE: Main Condition	RANGE: Other Conditions If Any
5a	Attack	Active	$\lvert\phi\rvert > \pi/2$	$V > V^*$	—	—	—	$R \geq S(V^*)$	—
5b				$V \leq V^*$	—	—	—	—	—
6				—	—	—	—	$R < S(V^*)$	—
7		Lost	—	$V > V^*$	—	—	—	—	—
8			—	$V \leq V^*$	—	—	—	—	—
9		Pass. No Act.	—	$V > V^*$	—	—	—	—	—
10			—	$V \leq V^*$	—	—	—	—	—
11	Evade	Active	$\lvert\phi\rvert \leq \pi/2$	$V > V^*$	—	—	—	—	—
12				$V \leq V^*$	—	—	—	—	—
13a			$\lvert\phi\rvert > \pi/2$	—	—	—	—	—	—
13b		No Active	—	—	—	—	—	—	—

Figure 4.7-1 Tactical Cases (m States) (Cont.)

RESPONSE FOR AIRCRAFT i

Case #(1) (m State)	Acceleration	Acceleration Bounds(2)	Turning Rate(3)	Turning Rate Bounds(2)	Comments
5a	$a = a(P_S)$	——	$\lvert\dot{\beta}\rvert = \dot{\theta} + \frac{\alpha - \eta}{\Delta t}$	$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))$	Not yet the time to slow down
5b	↓	——	↓	$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))\frac{V}{V^*}$	Not yet the time to slow down
6	$a = a_{DEC}$	——	↓	$\lvert\dot{\beta}\rvert \leq \dot{\beta}(g(V))$	Slow down to avoid overshoot
7	↓	$a \leq a(P_S)$	$\lvert\dot{\beta}\rvert = \dot{\beta}(G(V))$	——	Slow down to V*
8	$a = a(P_S)$	——	$\lvert\dot{\beta}\rvert = \dot{\beta}(g(V))\frac{V}{V^*}$	——	Speed up to V*
9	$a = a_{DEC}$	$a \leq a(P_S)$	$\lvert\dot{\beta}\rvert = \dot{\beta}(G(V))$	——	Slow down to V*
10	$a = a(P_S)$	——	$\lvert\dot{\beta}\rvert = \dot{\beta}(g(V))\frac{V}{V^*}$	——	Speed up to V*
11	↓	——	$\lvert\dot{\beta}\rvert = \dot{\beta}(g(V))$	——	Speed up if possible
12	↓	——	$\lvert\dot{\beta}\rvert = \dot{\beta}(g(V))\frac{V}{V^*}$	——	Speed up at least to V*
13a	↓	——	$\dot{\beta} = 0$	——	Speed up all the way
13b	↓	——	↓	——	Speed up all the way

Figure 4.7-1 Tactical Cases (m States) (Cont.)

FOOTNOTES TO FIGURE 4.7-1

(1) The cases are more finely broken down here than in the text. With one exception the conditions and doctrines are nonetheless the same, i.e., 13a and 13b both correspond to the discussion of 13. The exception is case 1b2, which results in a distinct doctrine from the rest of case 1; see Section 5.5.3.

(2) If the doctrine acceleration or turning rate is outside the limits of these columns, then the limit is used.

(3) On non-pursuit the sign of $\dot{\beta}$ is not given. The sign gives the direction of turn (left or right). This is determined by additional considerations; see Section 4.7.5.

(4) The acceleration a is first evaluated from case 1b1 and then this velocity criterion is checked.

(5) The value of $g(V)$ is set to $G(V)$ if $g(V) > G(V)$.

(6) The symbol $a(P_S)$ means the acceleration associated with the specific power function at the present speed and turning rate, see Section 4.6.2.1.

(7) While $a \leq a(P_S)$ is a universal restriction on acceleration, it must be specially checked here. Since $\dot{\beta}$ is restricted here by $G(V)$ rather than $g(V)$, $a(P_S)$ may be negative.

(8) The symbol $\exists y$ or $\nexists y$ means that a steady state speed y^* does or does not exist, respectively. See Section 4.7.1.1.

(9) Even if a steady state speed exists, the aircraft may not be able to turn as rapidly as the target at this apeed.

(10) The index i is suppressed in this table, but the opponent ia indexed by j.

it must slow down. If the range $R \geq S(V^*)$ (Case 5) it has time to speed up. Note that if $V < V^*$ then $S(V^*)$ is negative and the aircraft will naturally speed up.

When behind the target, the target speed, as noted, is relevant. However, only tha projection of the target speed on the line of sight is appropriate; if the pursuer is not directly off the tail of the target, then it will overshoot if it flies as fast as the target. Ideally, when $R = R^*$, the range rata of change, $\dot{R}$, is zero. From equation (4.3.1-1)

$$V_P \cos \alpha_P = V_T \cos \phi_P \quad , \quad \text{when } \dot{R} = 0 \quad .$$

Define

$$V_o = \frac{V_T \cos \phi_P}{\cos \alpha_P} \quad .$$

If the pursuer flies at speed V_o, $\dot{R}$ will be zero. If $V > V_o$ and $R < R^* + S(V_o)$ (Case 1), then overshoot will result and the aircraft must slow down. If $V > V_o$ and $R > R^* + S(V_o)$ (Case 2), then there is time to speed up. The former case, overshoot, forms an exception to the rule of never exceeding $g(V)$. With the danger of approaching the target too fast, the pursuer need not worry about maintaining speed, and instead will turn as hard as necessary to keep up with the target.

If $V < V_o$ and $R > R^*$ (Case 3), then clearly positive acceleration is indicated, just to get to R^*. The final case of pursuit, $V < V_o$, $R \leq R^*$ (Case 4), is more complicated. Here the pursuer has essentially attained its advantageous position, and must seek to preserve it. The target is going to maneuver also.

4.7.1.1 Steady State

Suppose the target cannot maintain a turn as tight as can the pursuer. Then the pursuer can arrive at a "steady state," in which so long as the target stays on a circular path, the pursuer can fly at some slower speed in a tighter circle, continually behind the target on pure pursuit. If the target flies straight, then the steady state is quickly achieved once the angle-off is zero, and the pursuer simply adjusts its speed to equal the target's. If the target is turning, the steady state speed y^* must be evaluated (see Figure 4.7-2).

For steady state the pursuer must turn at the target's turning rate $\dot{\beta}_T$. From the geometric construction

$$R_1^2 = R^{*2} + R_2^2 \quad , \quad \text{and}$$

$$V_T = R_1 \dot{\beta}_T \quad \text{or} \quad R_1^2 = \frac{V_T^2}{\dot{\beta}_T^2} \quad .$$

Combining, $V_T^2 = \dot{\beta}_T^2 R^{*2} + \dot{\beta}_T^2 R_2^2$. But $\dot{\beta}_T R_2$ is the desired velocity y^*, so

$$y^* = \sqrt{V_T^2 - (R^* \dot{\beta}_T)^2} \quad . \qquad (4.7\text{-}2)$$

The derivation assumes the target holds its turning rate constant. If the target changes course, y^* will change. Appendix F elaborates on the steady state requirements.

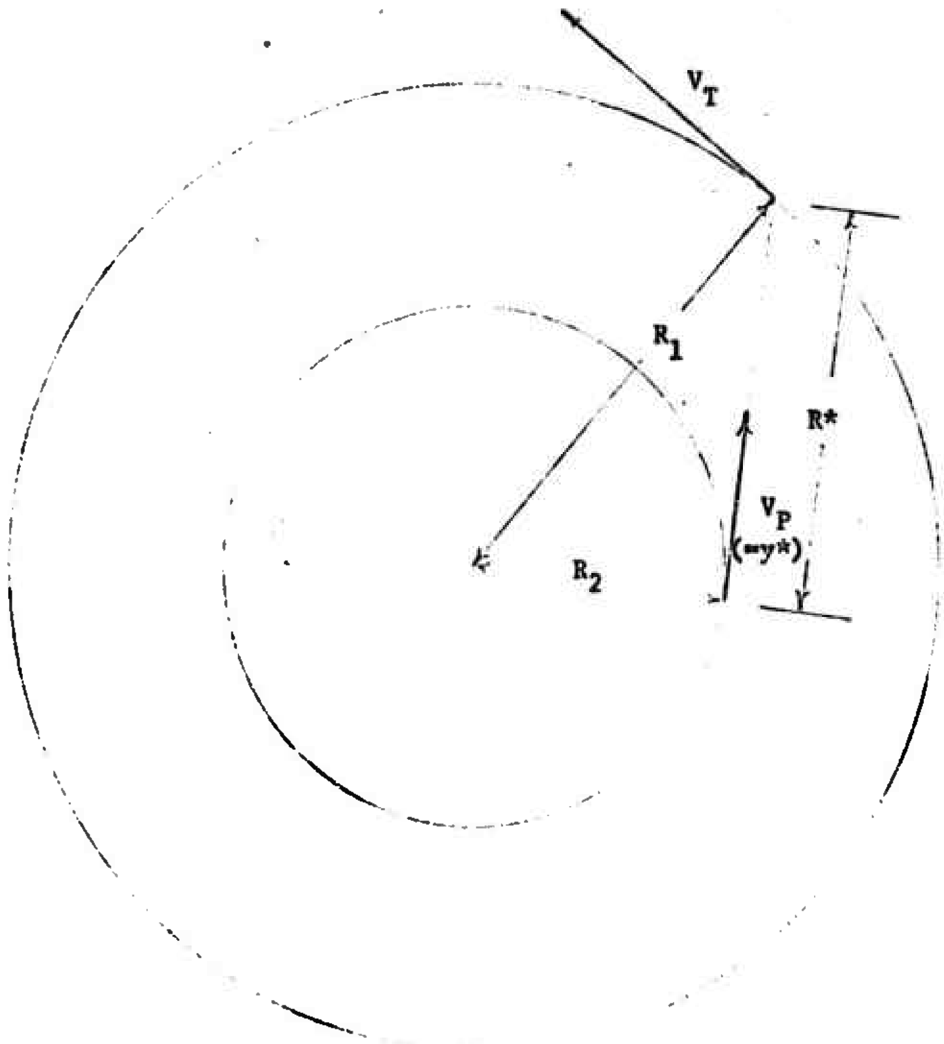

Figure 4.7-2 Steady State Geometry

If the aircraft can obtain this steady state (Cases 4a, 4b), then it should accelerate or decelerate to y^* . But steady state may be impossible (4c, 4d, 4e) either because the pursuer cannot turn as fast as the target, or because the ideal firing range R^* is too large. From Figure 4.7-2 if $R^* > R_1$ then the aircraft cannot fly inside the target circle. In such cases, the aircraft should not fly faster than the best turning speed V^* . If it is flying faster than this, it decelerates (to improve its turning rate). If slower, it accelerates to V^* <u>if the target can outmaneuver it at the present speed</u> $(\dot{\beta}_T > \dot{\beta}(g(V)))$. Otherwise it keeps the speed constant. This last is noteworthy. If the pursuer in this case tries to speed up to V^* , it will spiral outwards away from the target.

4.7.2 Non-Pursuit Doctrine with Good Information

So much for pursuit. Now suppose the aircraft obtains its target on the radar screen (or sees it visually), but is not yet on the pursuit course, or, because of the doctrine limitations on turning, $(g(V))$ or structural limitations, $(G(V))$ it cannot hold the pursuit. In such cases the aircraft will still find itself in one of the cases (1, 2, 3, 4, 5, 6) described above. It flies at the maximum g's allowable for the given case, until, hopefully, it arrives at pursuit.

However, if the same rules for maximum g's were used as are used on pursuit, a problem would arise. If positive acceleration is the doctrine of the given case, and the doctrine g loading were $g(V)$, then the aircraft would fly at a constant speed in a circle. Except in Case 4, in which the aircraft is in a controlling position, the target would likely get away, quite possibly to turn against the pursuer. Clearly a reduction in turning is often necessary so that the aircraft can speed up. But the amount of reduction is uncertain; the rule developed is as follows:

1. If $V \geq V^*$, then hold the constant circle at $\dot{\beta}(g(V))$ and do not accelerate. It is likely that the reduction of the ability to turn will prove too costly.

2. If $V < V^*$, then hold the turn to $\dot{\beta} = \frac{V}{V^*}(\dot{\beta}(g(V)))$. (4.7.2-1)

This will allow acceleration up to V^*.

4.7.3 Tactics with Poor Information

An aircraft may not have good information on the opponent. The bomber, for example, at the beginning of the engagement may not even be sura that there is an opponent, depending on the geometry at detection (Case 0). If the fighter comes in from tha rear, the bomber's radar may never pick it up. The bomber then could continue to fly atraight while the opponent starts firing. This condition is alleviated in two ways.

First, the model allows that once an opponent fires, the aircraft will become aware that an engagement is taking place, and, with little subtlety, will obtain IFF.[1]

4.7.4 Passive Information

Also, an aircraft may or may not have a passive radar receiver. This receiver has a pattern of reception over a circular sector, like the detection radar, but the pattern is off tha tail rather than the nose (see Figure 4.7-3).

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[1] For this reason the model further allows the fighter not to fire as soon as possible on an unaware target. The fighter may wait until the range initially set to R^* is reached so as to gain a better position before the target maneuvers.

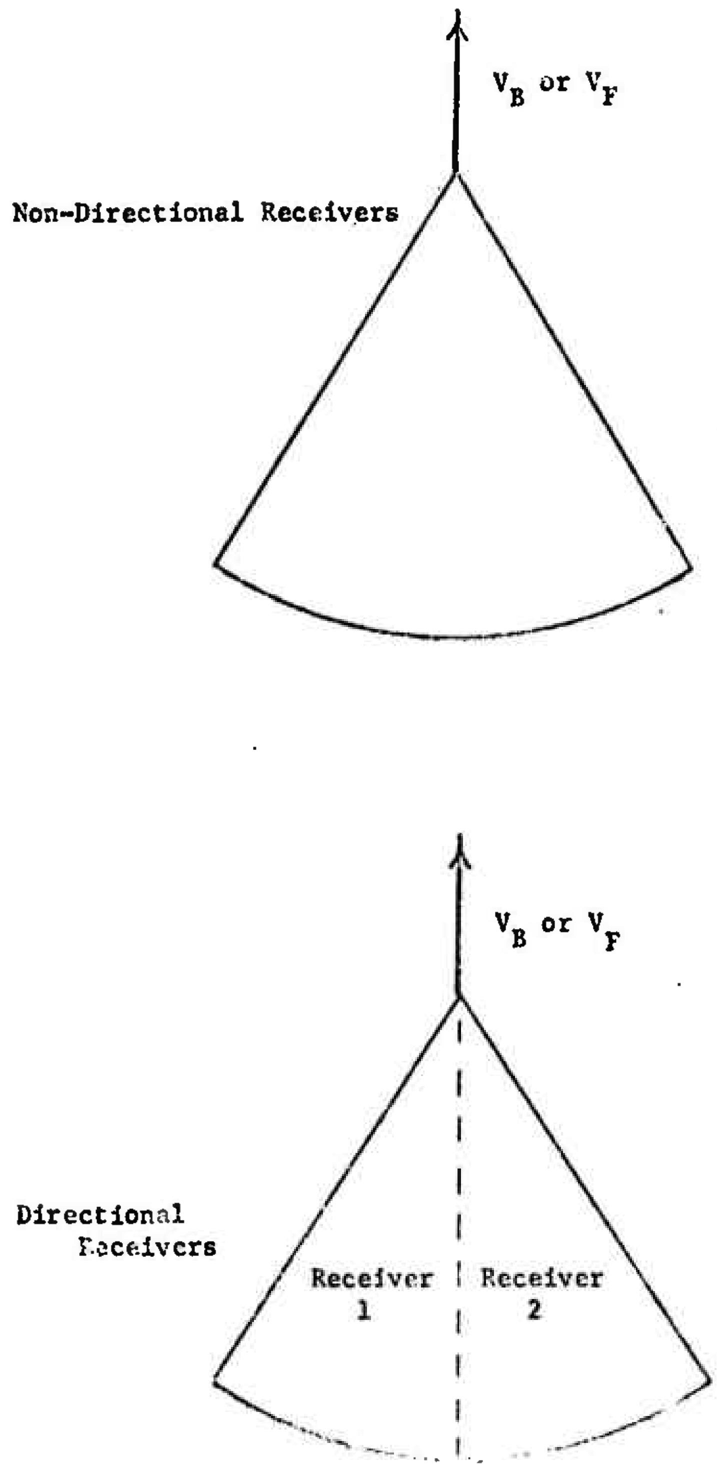

Figure 4.7-3 Passive Receivers

A passive receiver could pick up many insignificant signals, including periodic blips of the detection radars of possibly numerous other aircraft. An aircraft cannot react to every signal. The tracking radar, however, has a much higher scanning rate than a detection radar, and the model assumes that an unaware bomber becomes aware through its passive device only when the opponent's tracking radar is illuminating it.

To the fighter then, it is to his advantage not to turn on the tracking radar until necessary. The model reflects this with these options:

1. The fighter turns the tracking radar on for a moment only when firing a weapon. This is an idealized situation, to avoid considerations of time estimates and time of flight.

2. The fighter turns the tracking radar on shortly (a fixed time length τ) before it estimates it will begin firing, and then the radar is left on.

3. The fighter turns the tracking radar on as soon as detection occurs.

The passive radar in itself will not locate the opponent. However, the aircraft can be equipped with two such receivers so as to identify the side from which the opponent approaches.

4.7.5 Lack of Active Information

It is also possible that an aircraft was on pursuit but eventually lost the opponent on the detection radar. The standard doctrine is used: Turn into the opponent if possible (Rule 1). With no information it is assumed that a turn in the same direction as when information was available

(Rule 2) is most likely to be a turn into the opponent. If a non-directional passive radar receiver is the only source of information, the doctrine is to turn left (Rule 3). If a weapon firing alerts the bomber, the bomber turns right (Rule 4). The last two of these four rules are, of course, arbitrary. The first rule is natural; a turn in, rather than away, is generally safer, as the opponent has to turn harder.

The last two rules cause different results, depending from which side the opponent comes. Thus, to investigate the situation fully, the simulation is run for both cases (i.e., the model initializes the engagement both at ε and at $-\varepsilon$).

Acceleration rules for these "blind turn" cases are based on the assumption that the best defensive speed will be V^*, the speed for best sustained turning rate. If going faster than V^* (Cases 7, 9), decelerate and turn as hard as possible (up to $G(V)$). This is the other exception to the $g(V)$ rule. If $V < V^*$ (Cases 8, 10), then the convention of $\dot{\beta} = \dot{\beta}(g(V)) \frac{V}{V^*}$ applies, (equation 4.7.2-1).

4.7.6 Evasion

The underlying assumption on all doctrine up to now has been that an aircraft intends to destroy the enemy aircraft, or at least that the best defensive measure is to stay with the approaching aircraft and possibly convert on it. The model also allows the aircraft the option of attempting to leave the scene of combat either entirely or temporarily. Its decision to attack or evade is based on the information (active, passive, and optical) available.[1] If the aircraft knows it is behind the opponent it may turn away from the opponent, accelerate to maximum speed, and fly straight out. Otherwise, turning will probably prolong the engagement; so instead, it will immediately accelerate to maximum speed and fly straight. The precise formulation is: Suppose $\phi < 90°$ and the detection radar or optical system can find the opponent. If $V > V^*$ (Case 11), then maintain speed and fly at $\dot{\beta}(g(V))$. If $V < V^*$ (Case 12), then increase speed and fly at $\dot{\beta}(g(V)) \frac{V}{V^*}$.

For all other conditions (Casa 13) $\dot{\beta} = 0$ -- fly straight and accelerate to maximum velocity.

This concludes the discussion of acceleration and turning doctrine, summarized in Figure 4.7-1. Many of the rules are subjective and could be modified in the future.

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1) For example, the rule might be that with no information on the opponent, or with passive only, evade, but otherwise attack. When totally lost but nevertheless having established IFF, the model allows a choice of evasion depending on which aircraft can turn better; the aircraft did have information about the opponent previously, and it is assumed it could, at the time, size up the opponent aircraft this well.

SECTION 5

ENGAGEMENT MODEL
FLOW CHART DISCUSSION

5.1 Introduction

The model described in Section 4 is laid out in a flow chart form in Volume III. When modifying a model to satisfy computer needs many details arise that previously could be ignored. A prominent one is the measurement of angles. In Section 4, the angles 0°, 360°, or 720° are equivalent trigonometrically, and therefore, the differences are irrelevant. In the computer program it is important to limit the angles. If an angle is supposed to be between - 5° and 5°, then it would be a nuisance to check, (- 5°, 5°), (355°, 365°), etc. Similarly, if a derivation required only cosine, then θ and $-\theta$ are equivalent and the distinction is ignored. For computational purposes, however, some consistent rules are needed for angle definition. The first six diagrams of this section define the necessary angles. Figure 5.2-2 is repeated from Figure 4.3-1.

Other details arising in the flow chart include taking care of singular cases ignored in the model development, and giving precise formulation to certain concepts described in the model development. Finally, devices are used to speed up computer time.

This discussion will aid a reader already acquainted with the conceptual model in following the flow charts. It is to be read in conjunction with the flow charts. All symbols used are listed in Section 7.3.

5.2 General Conventions

5.2.1 Diagrams of Angles

These diagrams are for reference while reading the flow charts. To describe angles, it is first necessary to define conventions on the line of sight, Figure 5.2-1.

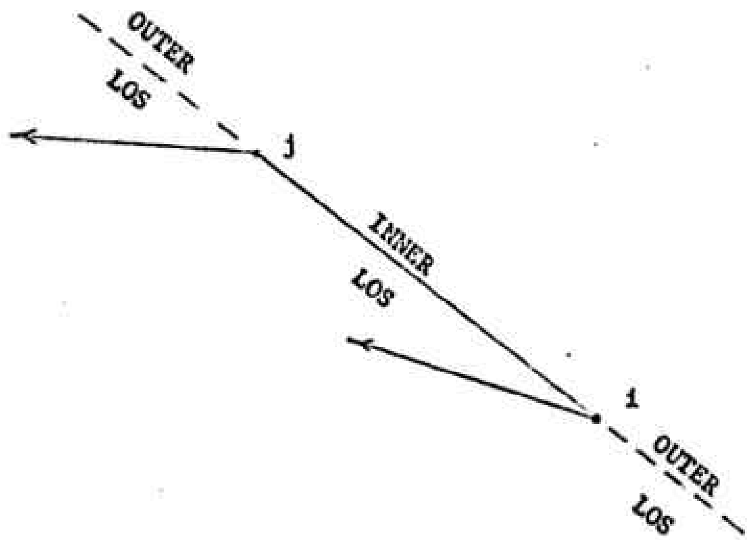


Figure 5.2-1 Line of Sight (LOS) Segments

- Figure 5.2-2 shows ths following angles:

  α_i : measured <u>from</u> the inner LOS <u>to</u> the velocity heeding of aircraft i .

  ϕ_i : measured <u>from</u> ths outer LOS <u>to</u> the velocity heading of aircraft j , j being the other aircraft.

  β_i : measured <u>from</u> the velocity heading of aircraft i <u>to</u> the positive x-axis (translated to head of ths vector $\vec{V}_i$).

There is a relationship between α_i and ϕ_j : $|\alpha_i| = \pi - |\phi_j|$

$\alpha_i > 0$: j is positioned to the right of i .

$\phi_i > 0$: i is positioned to the left of j .

$\dot{\beta}_i < 0$: i is turning counterclockwise.

All angles are measured counterclockwise and in the interval $[-\pi, \pi]$.

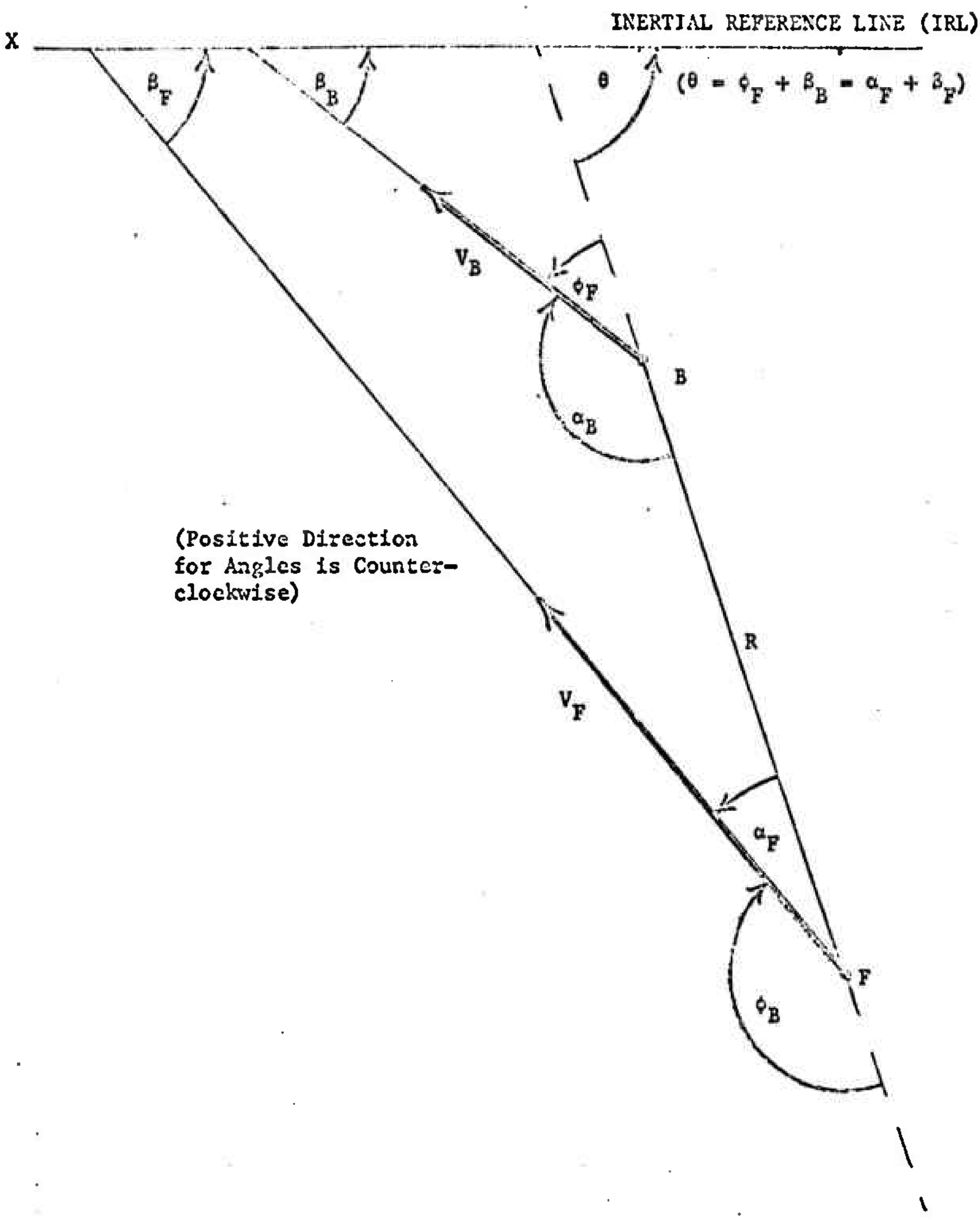


Figure 5.2-2 Defining Relative Positions

• Figure 5.2-3 describes the angle β .

$$\beta = \text{Arc}\tan\left(-\frac{dy}{dx}\right) ,$$

$$\beta_1 = \begin{cases} \beta , & \text{if } dx \le 0 \\ \beta - (\text{sgn } \beta)\,\pi , & \text{if } dx > 0 \end{cases} .$$

• Figure 5.2-4 shows various sensor angles. It should be noted that the magnitude of the angles need not follow in the order shown, e.g., $\alpha_{IFF}(i)$ could be greater than $\alpha_{TRK}(i)$.

• Figure 5.2-5 shows the meaning of the input angles in the weapon envelopes or firing tables. If the target aircraft is turning, or if the vulnerability of the target is not identical on each side, then it is essential to know which side of the aircraft an input refers to.

• Finally, Figure 5.2-6 describes the GRID coordinate system for initializing engagements. Note that the angle ε is measured <u>from</u> $\vec{V}_F$ <u>to</u> $\vec{V}_B$.

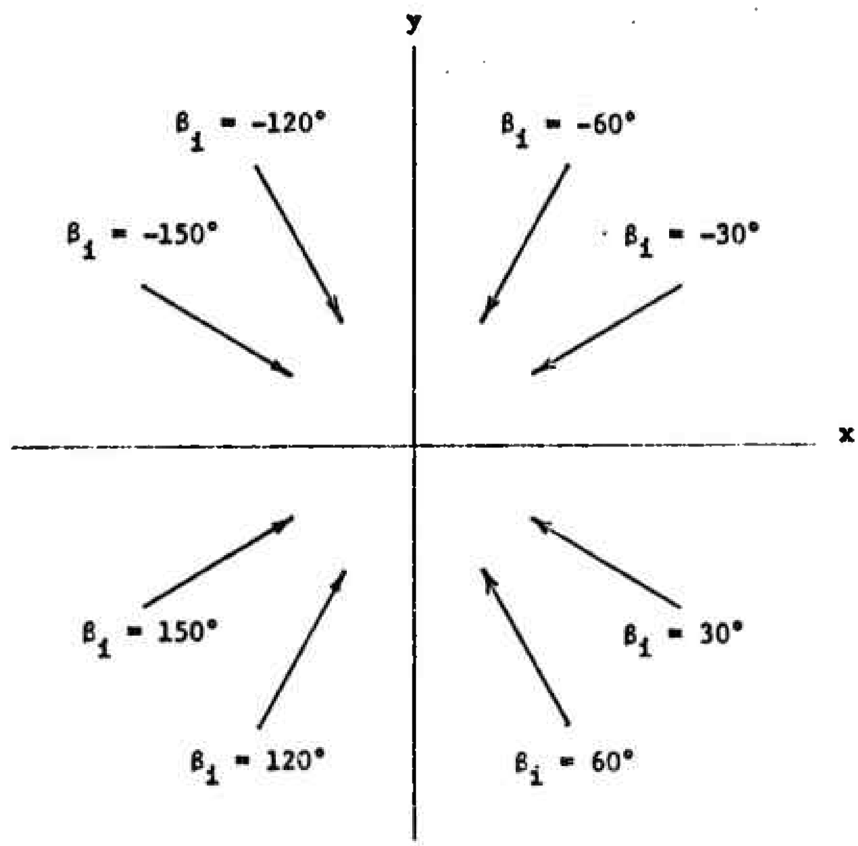


Figure 5.2-3 Orientation of β_i with Respect to the Inertial Coordinate System

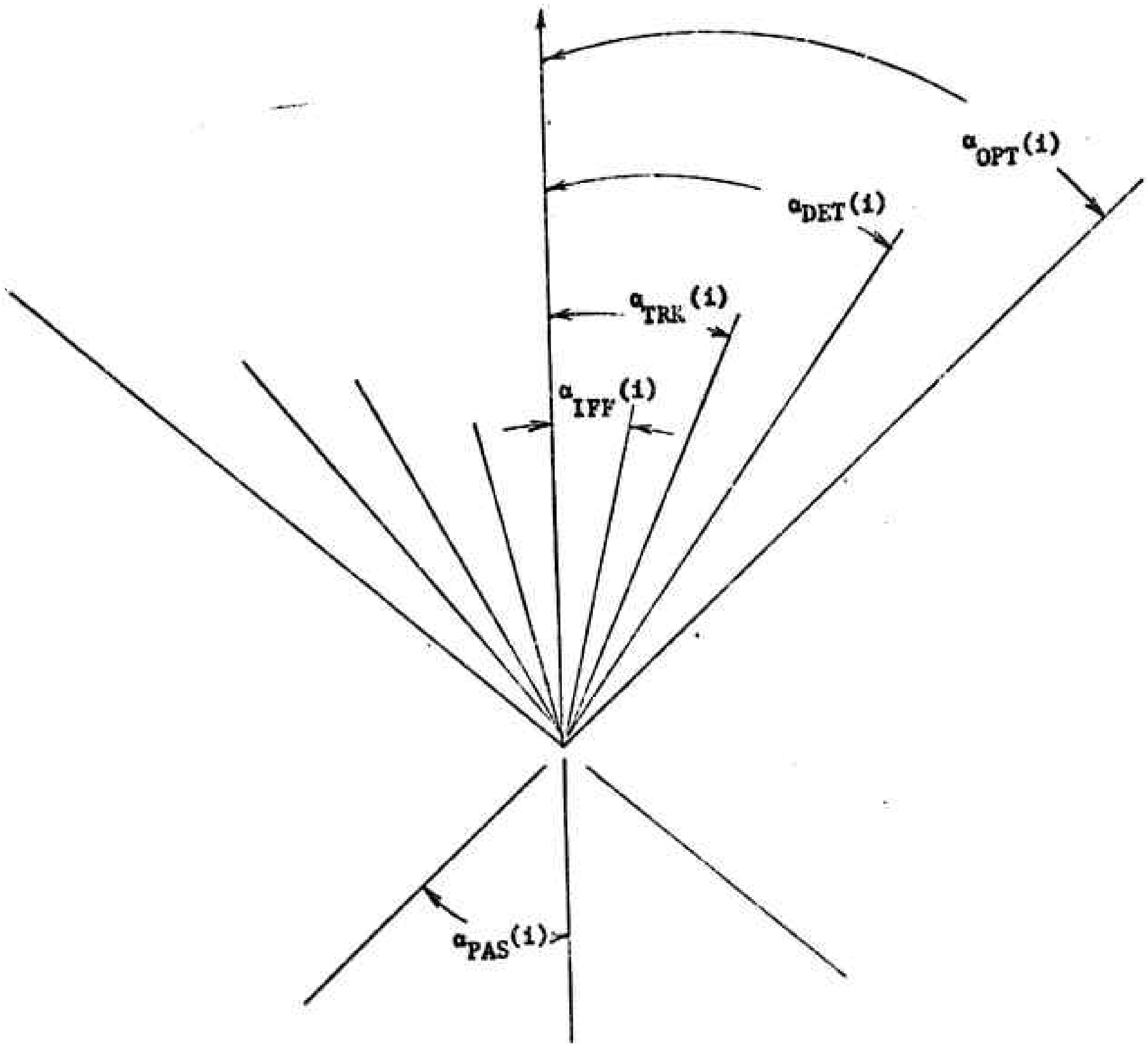


Figure 5.2-4 Aircraft Sensor Half-Angles

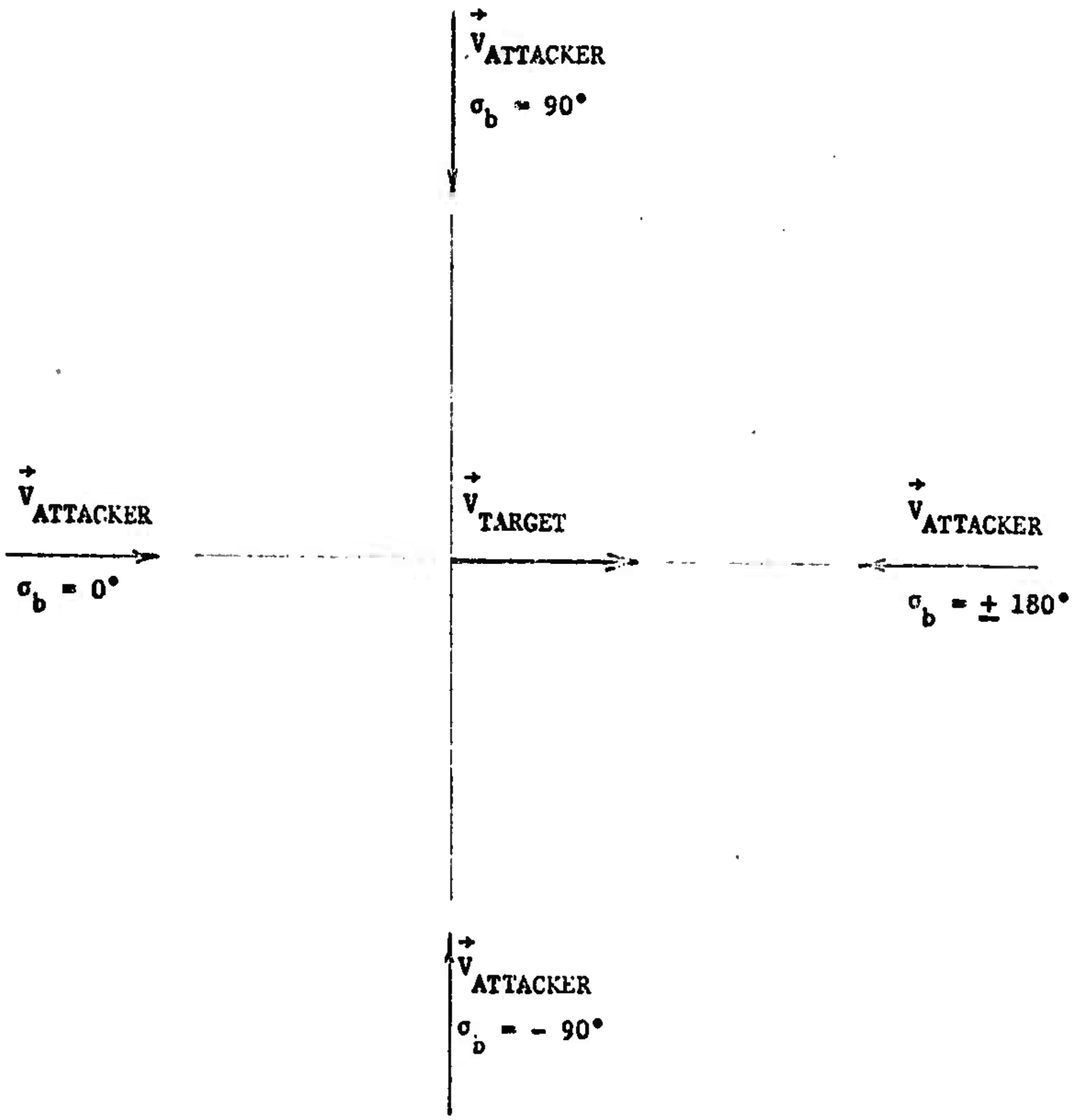


Figure 5.2-5 Orientation of σ_b for Firing Tables

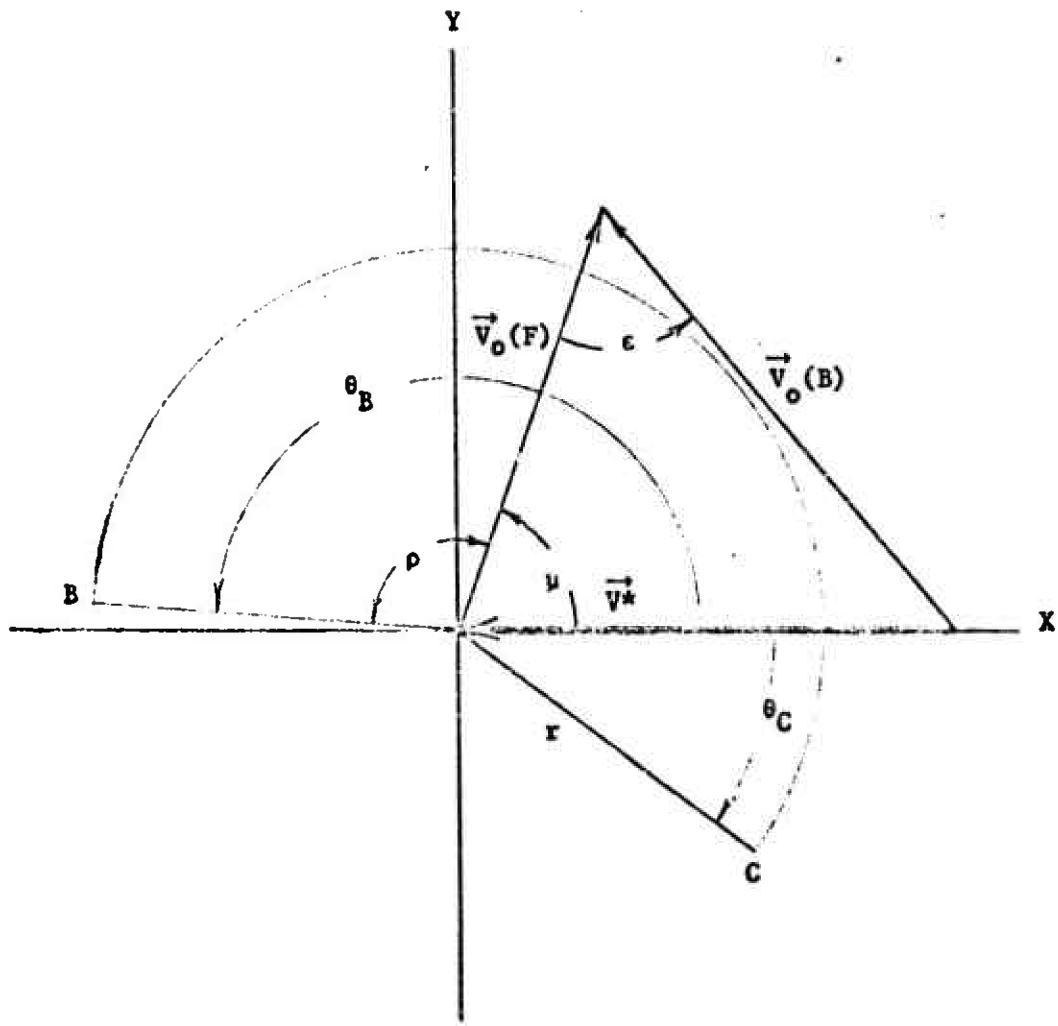


Figure 5.2-6 Definition of Angles in the GRID (X, Y) Coordinate System

5.2.2 Flow Chart Conventions

Flow charts have been provided in Volume III, Section 8 to support the descriptions of the various programs which comprise ATAC-2. The conventions and symbols used in these flow charts are presented below.

Rectangles of any size indicate a processing function; usually refer to evaluating a variable by some mathematical expression but also used to refer to a collection of processing functions as represented by a routine or subroutine.

Hexagonal figures of any size indicate a decision function involving a question; a program branches to an appropriate instruction depending upon the answer to the question. The symbol always implies a question although a question mark is never included. For example, this symbol used to enclose the statement $\dot{\beta}_i = 0$ means "Is $\dot{\beta}_i$ equal to zero?"

START

Indicates the point at which a program, routine or subroutine begins.

RETURN

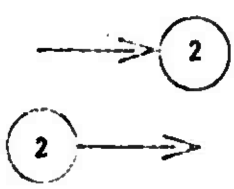
Used at a point in a routine to indicate that when this point is reached program control is to be returned to the program which executed (called) the routine or subroutine.

$\longrightarrow$

Indicates the flow of processing.

$\beta_i = \beta_i + \dot{\beta}_i \Delta t$

FORTRAN instead of mathematical statements are used throughout. For example, the statement on the left is in FORTRAN and indicates that the new value of β_i is obtained by adding $\dot{\beta}_i \Delta t$ to the old value of β_i.



Used to indicate the flow of processing by a disconnected line when a continuous line would involve cross-overs of other lines of flow; flow to a circle at the head of an arrow is resumed at the circle at the tail of an arrow containing the same number as the first circle.

5.3 The EXECUTIVE Routine - EM

The EXECUTIVE Routine - EM (see Volume III, page 24) essentially controls the execution of the main routines of the ENGAGEMENT Model. It initializes the values of certain parameters and calls programs in their proper sequence. Specifically, the EXECUTIVE Routine performs the following functions:

(1) calls the INPUT Routine,

(2) determines the first value of ε, the initial crossing angle of the aircraft to be used,

(3) determines the active detection capability of the fighter,

(4) calls the GRID PREPARATION and COMBAT programs, and

(5) increments the value of ε.

The INPUT Routine is executed first. It reads the values of the input variables into computer storage and performs other functions such as converting angles input in degrees to radians.

The active detection capability of the fighter which is used to establish the point at which the fighter initially detects the bomber is determined by the EXECUTIVE Routine - EM. The variables r (range) and ρ (the half-angle) define this capability. These variables are assigned the values of the corresponding parameters of a fighter's detection radar equipment if it has such equipment, otherwise of its optical capability. Usually if a fighter has both radar and a visual capability, the radar range will be greater than the visual range. If the angular coverage of the visual capability exceeds that of the radar, the procedure discussed above will ignore the extra visual coverage. It is believed, however, that this procedure for most cases of interest should not materially distort the distribution of points at which initial detection can occur. It should be noted that the full optical capability specified in the inputs is used in all other operations of the model.

In some cases it may be desirable to reduce the detection range in the search phase. An input r_{in} allows for this. After initial detection, R_{DET} and R_{OPT} are used, irrespective of r_{in}.

The EXECUTIVE Routine - EM selects the values of ε to be used for a given run of the ENGAGEMENT Model. A run can be made for one value or a set of values of ε. If only one value is to be considered the operator must set $\Delta\varepsilon$ to a value greater than 180° and assign ε the value desired. EXECUTIVE will then execute the ENGAGEMENT Model for this value of ε only. If $\Delta\varepsilon$ is assigned a value less than 180°, EXECUTIVE will generate the set of values to be used for ε. Two cases involving three conditions of each aircraft can arise in this situation. The fighter or bomber may have

(1) no passive detection capability,

(2) passive detection capability with side discrimination, or

(3) passive detection capability without side discrimination.

Under condition (1) an aircraft will follow a linear flight path until fired upon or until the enemy aircraft is actively detected, and then will attempt to turn into the enemy. Under condition (2), an aircraft will turn into the enemy aircraft when passive or active detection information is received. Under condition (3), an aircraft will execute a tactical doctrine turn to the left when passively detecting the enemy aircraft regardless of the side from which an enemy is approaching. The first case arises if any combination of only conditions (1) and (2) apply to the fighter and bomber. The second case arises if condition (3) applies to either the fighter or the bomber. In the first case, all outcomes of a simulation will be symmetric with respect to ε ; that is, the same outcomes will occur for ε in the interval [- 180°, 0] as the interval [0, 180°].[1] Therefore, EXECUTIVE generates a set of ε's only in the interval [0, 180°] in the first case, in order to eliminate unnecessary simulations. In the second case, outcomes will not be symmetric with respect to ε , so EXECUTIVE generates a set of ε's in the interval [- 180°, 180°].

After the above functions of the EXECUTIVE routine have been performed the major loop of this routine is entered. This is the loop in which ε is incremented. For each value of ε the GRID PREPARATION and COMBAT routines are executed once. However, within the COMBAT routine, for each value of ε , there will be N engagements run corresponding to the N grid-points, (see discussion of GRID PREPARATION, Section 5.4 and GRID, Section 5.5.1). Finally, when ε reaches a value greater than π the program stops ending a completed set of simulated engagements.

5.4 The GRID PREPARATION Routine

All simulated engagements in ATAC-2 are initiated at a time when the bomber first enters the fighter's active (radar or optical) detection pattern. For each ε , the initial angle between velocity vectors of the

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1) In the particular instance that the bomber is first alerted by being fired on, asymmetric results occur, since it will make a doctrine right turn. But this instance was not deemed of sufficient moment to be treated as Case 2.

two aircraft, GRID PREPARATION (see Volume III, page 25) in conjunction with the GRID routine of the COMBAT program generates N equally likely points along the perimeter of the fighter's active detection pattern at which detection could occur.

The fighter's active detection pattern is taken to be a sector of a circle. The bomber's path will intersect this pattern along the relative velocity vector defined by ε and the velocities of the two aircraft. Intersection can occur along the arc or radial portions of the detection pattern. The procedure employed here essentially determines the points at which N equally spaced paths parallel to the relative velocity vector, $\vec{V}^*$, will first intersect the fighter's active detection pattern. These points of intersection are referred to as grid-points.

Proceeding from one side of a detection pattern to the other, the N paths can intersect the pattern in many different sequences of arc and radial segments depending upon the included angle of the circular sector and its orientation with respect to the relative velocity vector. For example, consider the case illustrated in Figure 5.4-1. The N relative bomber's paths can intersect the fighter's active detection pattern in the arc "Y_{max} to C," in the radial segment "C to F," in the radial segment "F to B," and in the arc "B to Y_{min}." (The point B has no relation to the bomber location.) A rectangular coordinate system (X, Y) has been defined in which computations are performed to locate the N points of intersection. The origin of this system is the fighter and the X-axis is parallel to $\vec{V}^*$ with its positive sense opposite the direction of $\vec{V}^*$.[1] The points

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1) The (X, Y) system used here should not be confused with the inertial coordinate system (x, y) used elsewhere in AIAC-2.

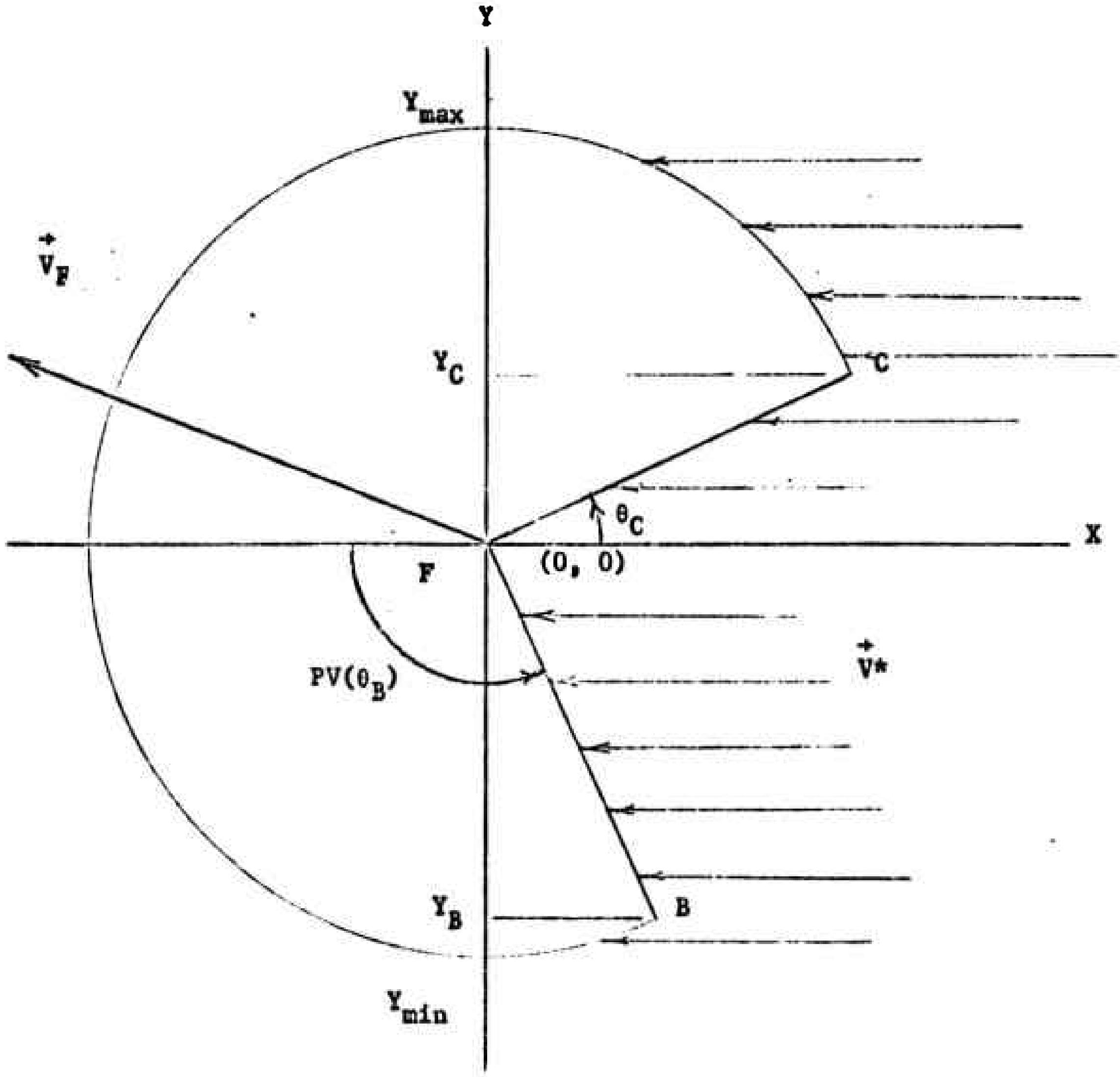

Figure 5.4-1 Logical Basis for GRID PREPARATION And GRID Routines

$(0, Y_{max})$, C, F (0,0), B and $(0, Y_{min})$ shown in Figure 5.4-1 represent the end points of different segments along the perimeter of the fighter's detection pattern in which initial detection can occur. Different equations are used to locate initial detection points within these segments. The quantities Y_C, 0, and Y_B are employed as control parameters to select the proper equation for locating each of the N initial detection points. GRID PREPARATION establishes the values of these quantities, the value of Y_g used by GRID to determine the initial grid-point and the value of the decrementing interval ΔY_g. It is executed once for each value of ε.

The GRID Routine uses these values to generate one of the N grid-points each time it is executed.

In the system devised for locating grid-points, 22 cases arise which require individual treatment. The 22 cases are presented pictorially in Figure 5.4-2. The ordered pair of numbers appearing in each cell of this figure refers to the quadrants (defined in the conventional manner) in which the points B and C, respectively, are positioned. The logic of the GRID PREPARATION Routine is designed to identify the case which applies to a particular set of conditions and then to set the control parameters so that GRID can properly locate each grid point. The case shown in Figure 5.4-1 is the case upon which this logic is based.[1] In other words, the logic assumes that as Y_g is decremented from Y_{max} to Y_{min} detection

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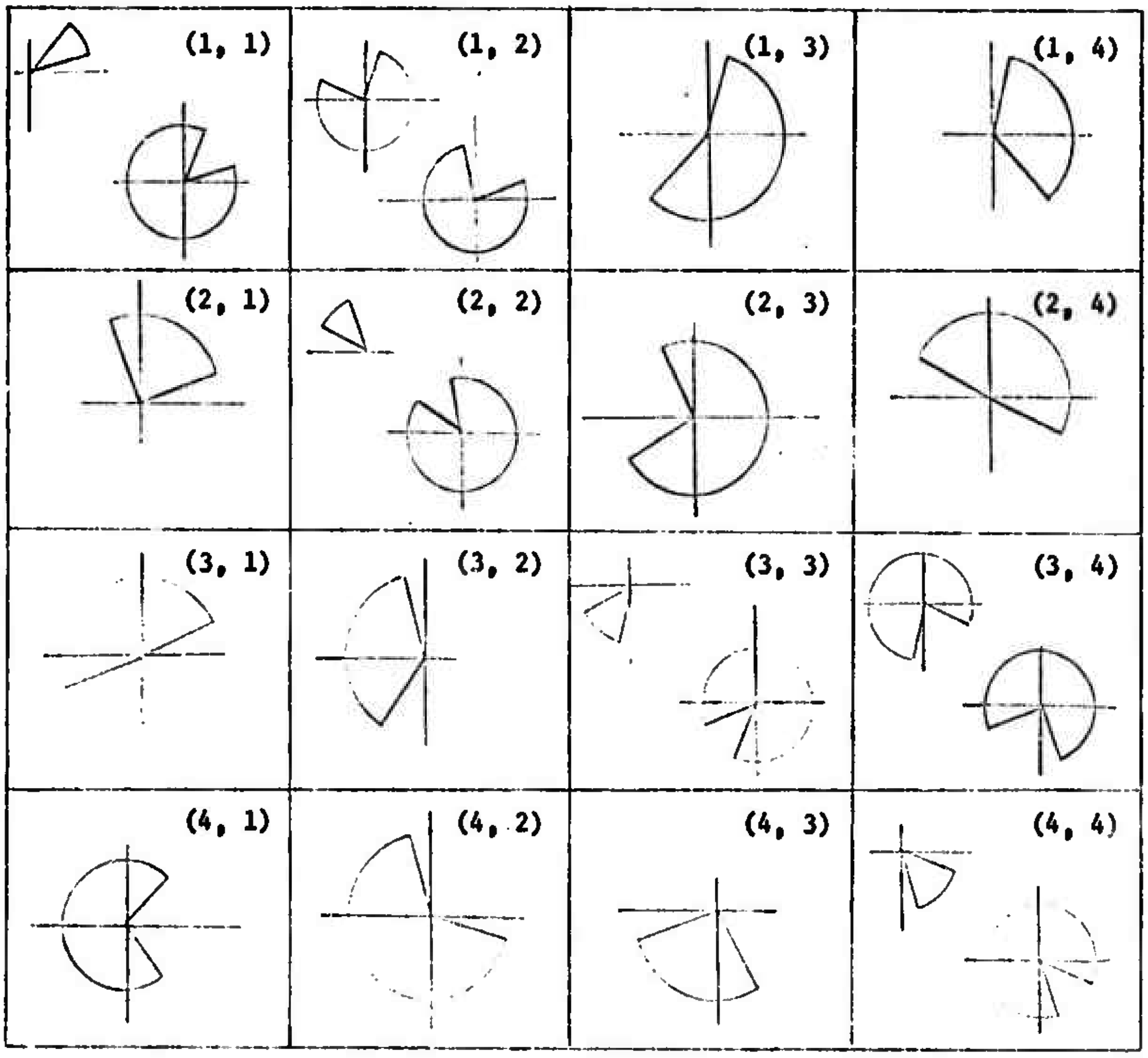
1) The use of this case as the logical basis for the GRID PREPARATION and GRID Routines is not intended to imply that this is the most typical case. On the contrary, it probably is an unusual case because it involves the condition $V_o(B) > V_o(F)$. It was chosen as the logical basis because it represents the extreme condition in which grid-points can be located along four segments on the perimeter of the fighter's active detection pattern.



NOTE: Ordered pair (a, b) denotes $Q_B = a$ and $Q_C = b$. Positive X-axis is directed to right and positive Y-axis upward. The point C is always clockwise around the arc from B.

Figure 5.4-2 CRID PREPARATION Cases - Possible Orientations of Fighter's Active Detection Pattern in X, Y System occurs first along the arc "Y_{max} to C," then along the radial segment "C to F," then the radial segment "F to B" and finally along the arc "B to Y_{min}." Not all of the program branches corresponding to these segments apply to all cases. GRID PREPARATION, therefore, sets the values of the control parameters so that the branches are appropriately suppressed or executed for each particular case.

A more formal treatment of the development of the GRID PREPARATION routine follows. Consider the rectangular coordinate system (X, Y) defined before to have as its origin the position of the fighter and its X-axis parallel to the relative velocity vector $\vec{V}^*$ with positive sense opposing the direction of $\vec{V}^*$. This coordinate system is shown in Figure 5.2-6. The vector diagram included in this figure indicates that $\vec{V}^*$ is defined with the fighter as the fixed aircraft. Applying the law of cosines to this vector diagram, yields the following expression for the magnitude of $\vec{V}^*$:

$$V^* = \left[V_o(F)^2 + V_o(B)^2 - 2V_o(F)\ V_o(B)\ (\cos\ \varepsilon)\right]^{1/2},$$

where $V_o(F)$ and $V_o(B)$ are the initial velocities of the fighter and bomber, respectively. V^* will be non-zero except for the case in which $V_o(F) = V_o(B)$ and $\varepsilon = 0$. In this case, the X-axis is taken to be parallel to, and has positive sense the same as, the direction of $V_o(F)$.

Define a line segment Y^* as the normal projection of the fighter's active detection pattern onto the normal to $\vec{V}^*$, that is, onto the Y-axis. Assume the position of the bomber is uniformly distributed in a space traveling parallel to $\vec{V}_B$. The fighter moves through this space with a velocity $-\vec{V}^*$. The fighter will detect the bomber if and only if the area swept out by the segment Y^* includes the position of the bomber. Given detection, the original uniform distribution of the bomber's position generates a uniform distribution of detection points along Y^*. Grid-points are selected according to this distribution of the bomber along Y^*. This is done by considering N intervals of length $\Delta Y_g = Y^*/N$ along the segment Y^* and placing the bomber's initial position on the fighter's active detection perimeter so that $\vec{V}^*$ bisects each of the N intervals.

The fighter's active detection capability is uniquely defined by ρ, the half-angle of coverage measured from the fighter's nose, and a range r.[1] Thus, the fighter's active detection pattern is a sector of a circle of radius r and included angle 2ρ. We define μ to be the angle between the positive X-axis and $V_o(F)$, and the angles θ_B and θ_C to be the angles between the positive X-axis and the two radial extremities of the fighter's active detection pattern. These three angles are measured positive in the counterclockwise direction from the positive X-axis. From the construction given in Figure 5.4-3,

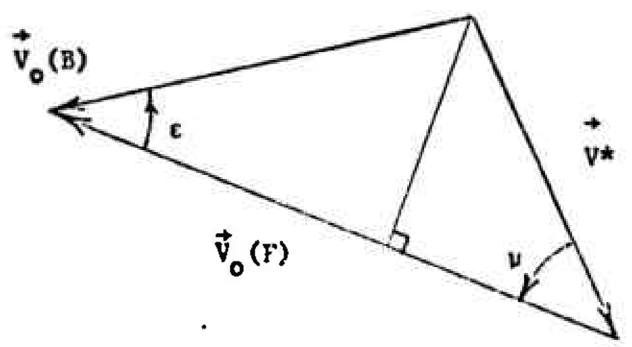


Figure 5.4-3 Determination of μ

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1) See Section 5.3, the EXECUTIVE Routine, for a more complete definition of ρ and r.

it can be observed

$$\mu = \text{Arc cos} \left(\frac{V_o(F) - V_o(B) \cos \varepsilon}{V^*} \right) .$$

In the event that V^* is zero, ε is set equal to 0. The Arc cosine function for the computer system utilized yields a value of μ in the range $[0, \pi]$. However, it is necessary to recognize negative as well as positive values of μ. It happens that μ always has the same sign as ε whose range is $[-\pi, \pi]$. Thus, the above expression is used to obtain the proper absolute value for μ and the sign of ε is tested to obtain the proper sign for μ.

The definitions of θ_B and θ_C are

$$\theta_B = \mu + \rho$$

and

$$\theta_C = \mu - \rho \quad ,$$

where ρ is always positive.

Expressions can now be developed for the control parameters Y_B and Y_C. Initially define Y_B and Y_C to be the Y-coordinates of the points B and C, respectively. Thus,

$$Y_B = r \sin \theta_B \quad ,$$

and

$$Y_C = r \sin \theta_C \quad ,$$

which can be established by referring to Figure 5.4-1. The quantities Y_{max} and Y_{min} are the Y-coordinates of the end points of the line segment Y^*, where $Y_{max} > Y_{min}$. These quantities can assume the values $\pm r$, 0, Y_B or Y_C depending upon the particular case being considered. For example, $Y_{max} = r$ and $Y_{min} = -r$ for the case given in Figure 5.4-1 whereas $Y_{max} = Y_B$ and $Y_{min} = Y_C$ for the case shown in Figure 5.4-3. If the detection pattern is entirely on one side of the X-axis either Y_B or Y_C must sometimes be redefined to zero.

In Figure 5.4-2 (1, 1) top left, Y_B is set to zero, since Y_{min} is below Y_C, and the program must be supplied with the information that the final grid-points are radial points. Consider (3, 1) to appreciate the necessity. The value of ΔY_g can be calculated from

$$\Delta Y_g = \frac{Y_{max} - Y_{min}}{N} ,$$

$$Y_{max} - Y_{min} = Y^* .$$

After determining the values of the control parameters, GRID PREPARATION computes a value Y_g from the expression

$$Y_g = Y_{max} + 0.5 \, \Delta Y_g .$$

This value is an input to GRID where it is immediately decremented by a full ΔY_g. This determines the Y coordinate of the mid-point of the first of the N intervals within the line segment Y^*.

The last step executed by GRID PREPARATION is the computation of a factor F from the expression

$$F = \frac{Y \; V^*}{V_o(B)} \quad .$$

F is a factor employed in the DATA PROCESSING Model to compute the probability of the fighter detecting the bomber. An explanation of this factor is given in Appendix A, Section A.2.

The GRID PREPARATION routine's logic shown in the flow chart follows the general explanation given above. The elaborate branching arrangement shown provides for uniquely identifying each of the 22 cases presented in Figure 5.4-2. Cases are identified by establishing the quadrants of the points B and C . For those instances in which two cases occur for B and C in the given quadrants, the identification involves comparing Y_B and Y_C . An understanding of the operation of any particular branch can be achieved by referring to Figures 5.4-1 and 5.4-2 and following the logic through to the GRID Routine.

5.5 The COMBAT Routine

The COMBAT Routine (see Volume III, page 26) constitutes the heart of the ENGAGEMENT Model of ATAC-2; it conducts the simulated engagements between two aircraft. COMBAT is basically a calling sequence which executes a collection of routines in the proper order to effect a series of simulated engagements. A series consists of the engagements initiated from the grid-points for a given initial crossing angle ε . In the operation of COMBAT the steps are essentially:

(1) selection of a grid-point,

(2) initialization of engagement conditions for the selected grid-point, and

(3) iteration of a basic cycle which conducts an engagement through a time pulse until terminal conditions are attained.

After the engagements for all grid-points for a given ε have been completed, COMBAT returns control to the EXECUTIVE Routine.

The first routine executed by COMBAT is GRID. Based upon the values of quantities determined by GRID PREPARATION, GRID determines the initial range between the two aircraft and the initial tracking angles which define a grid-point. The values of other engagement parameters and program control variables are initialized by the INITIALIZE FLIGHT Routine. At this point the first iteration of the basic cycle of a simulated engagement is executed.

NAVIGATIONAL SYSTEMS establishes the information state of an aircraft and then selects the maneuver which the aircraft will execute in a given time pulse. The maneuver is based upon the information state, engagement conditions and a tactical doctrine inherent in the ENGAGEMENT Model. During the basic cycle, this routine is executed twice, once for the fighter as the subject for a maneuver decision, and once for the bomber. ADVANCE RELATIVE COORDINATES then determines the incremental relative movement of the two aircraft. It uses the values of variables which define the aircraft's maneuvers as selected for the given time pulse by NAVIGATIONAL SYSTEMS. The new positions of the aircraft with respect to each other are established by ADVANCE RELATIVE COORDINATES from the determined incremental relative movement. TRANSFORM TO INERTIAL COORDINATES is then executed to establish the inertial coordinates (x, y) of each aircraft as the result of the movement in the given time pulse. FIND G_i determines the total g's which each aircraft pulls in performing the maneuver selected for the given time pulse. These values of total g's enter into the evaluations made by CHECK WEAPONS which is the next routine to be executed. In addition FIND G_i evaluates the oxygen debt of the pilot. CHECK WEAPONS evaluates whether any weapons can be fired. Whenever a weapon is fired, the time and the values of certain engagement variables are stored for eventual printout or use by the DATA PROCESSING Model. A running total of the time during which each weapon type's firing requirements are satisfied is also kept. PRINT causes the inertial coordinates of the fighter and bomber to be printed out if the simulation time has reached the end of a print interval t^*. The OVER Routine is executed after execution of the PRINT Routine to determine if the engagement conditions at the end of the given time pulse meet any of the stipulated terminal conditions. If they are not met, the program returns to NAVIGATIONAL SYSTEMS to perform another iteration of the basic simulation cycle. If they are met, weapon firing information and the total time of the engagement are stored by the RESULTS Routine. A test is then executed to determine if the grid-point, from which the engagement just concluded was initiated, was the last grid-point to be considered for the given ε. If this is so, control is returned to the EXECUTIVE Routine. Otherwise, the COMBAT Routine returns to the GRID Routine for selection of the next grid-point and a repetition of the process.

5.5.1 The GRID Routine

The GRID Routine (see Volume III, page 27) locates one of the N grid-points. The grid-point is located in relative coordinates in terms of the range between the two aircraft, R, and the tracking angle of the fighter, α_F , and bomber, α_B . To locate properly each of the grid-points, GRID uses the values of the control parameters Y_B , Y_C and ΔY_g which were assigned for the given ε by GRID PREPARATION.

GRID moves along the segment Y^* , the projection of the fighter's active detection pattern onto the Y-axis, from higher to lower values of Y_g .[1] Thus, it first decrements the value of Y_g to locate the mid-point of the next of the N intervals to be considered along Y^* . The value of an angle γ , shown in Figure 5.5-1, is calculated from the expression

$$\gamma = \text{Arc sin} \left[\frac{Y_g}{r} \right] .$$

This angle is used to locate grid-points which fall on the arc of the fighter's active detection pattern.

GRID then examines the value of Y_g to determine whether the grid-point corresponding to Y_g will fall along an arc or a radial segment of the fighter's detection pattern. The branch of the routine which will locate the grid-point on the proper segment of the pattern is then executed resulting in the calculation of the proper values of R , α_F and α_B . Each branch essentially determines the point on a segment of the pattern at which a vector drawn through the mid-point of an interval of Y^* intersects the segment. If the intersection occurs on an arc of the pattern, R will be equal to the detection range r , and α_F will be the principal value[2] of the quantity $(\nu - \gamma)$ as shown in Figure 5.5-1. If the inter-

---

1) See Section 5.4, GRID PREPARATION Routine, for a definition of the (X,Y) coordinate system and the overall grid-point selection procedure.

2) A value in the interval $[-\pi, \pi]$. See Section 5.5.11.1 for a description of the principal value function.

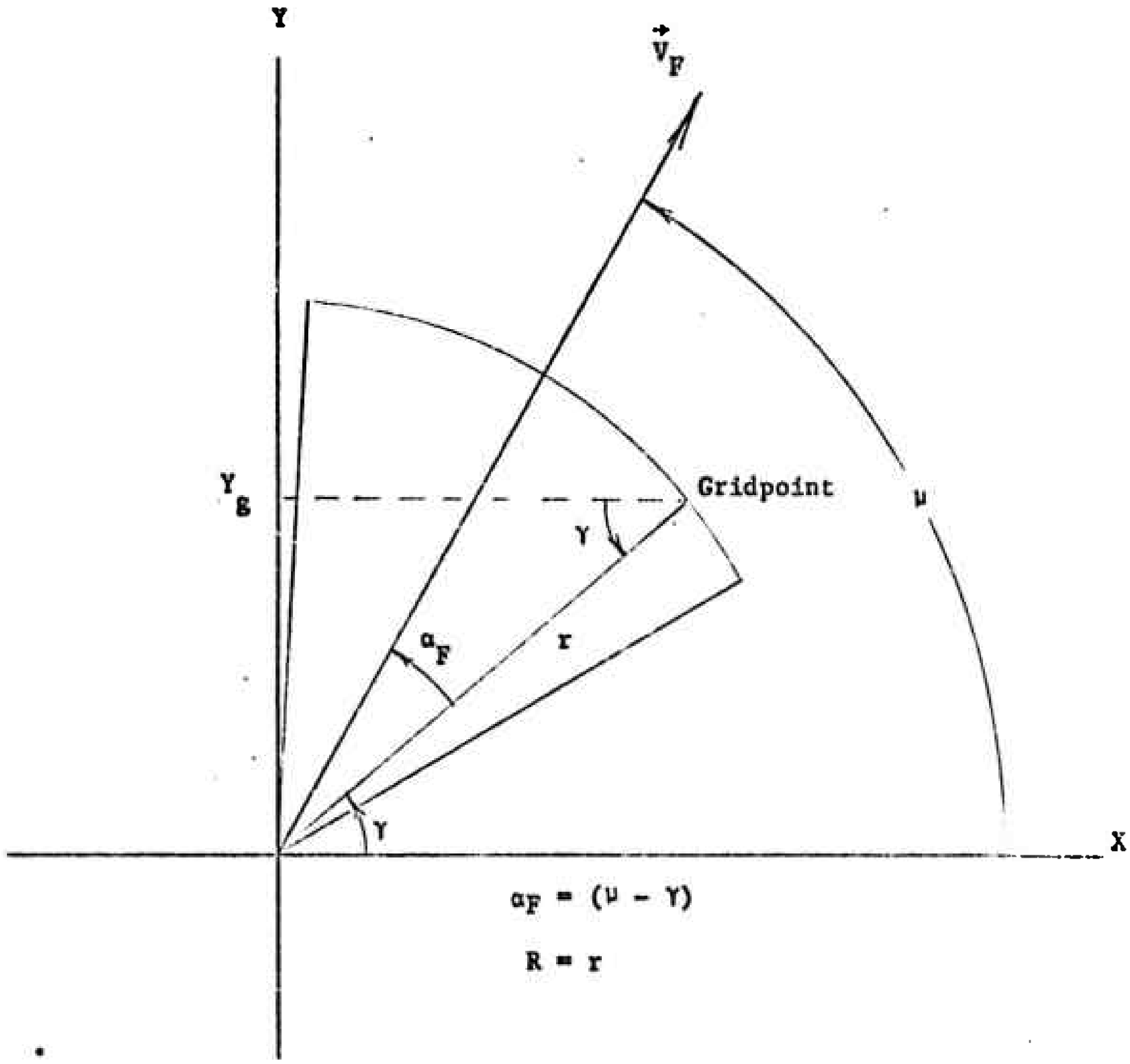


Figure 5.5-1 Determination of R and α_F for Gridpoint on Arc of Fighter's Detection Pattern

section occurs along a radial segment of the pattern, α_F obviously will be equal to ρ or $-\rho$, the detection half-angle, and R will be given by

$$R = \left| \frac{Y_g}{\sin (\mu - \alpha_F)} \right|$$

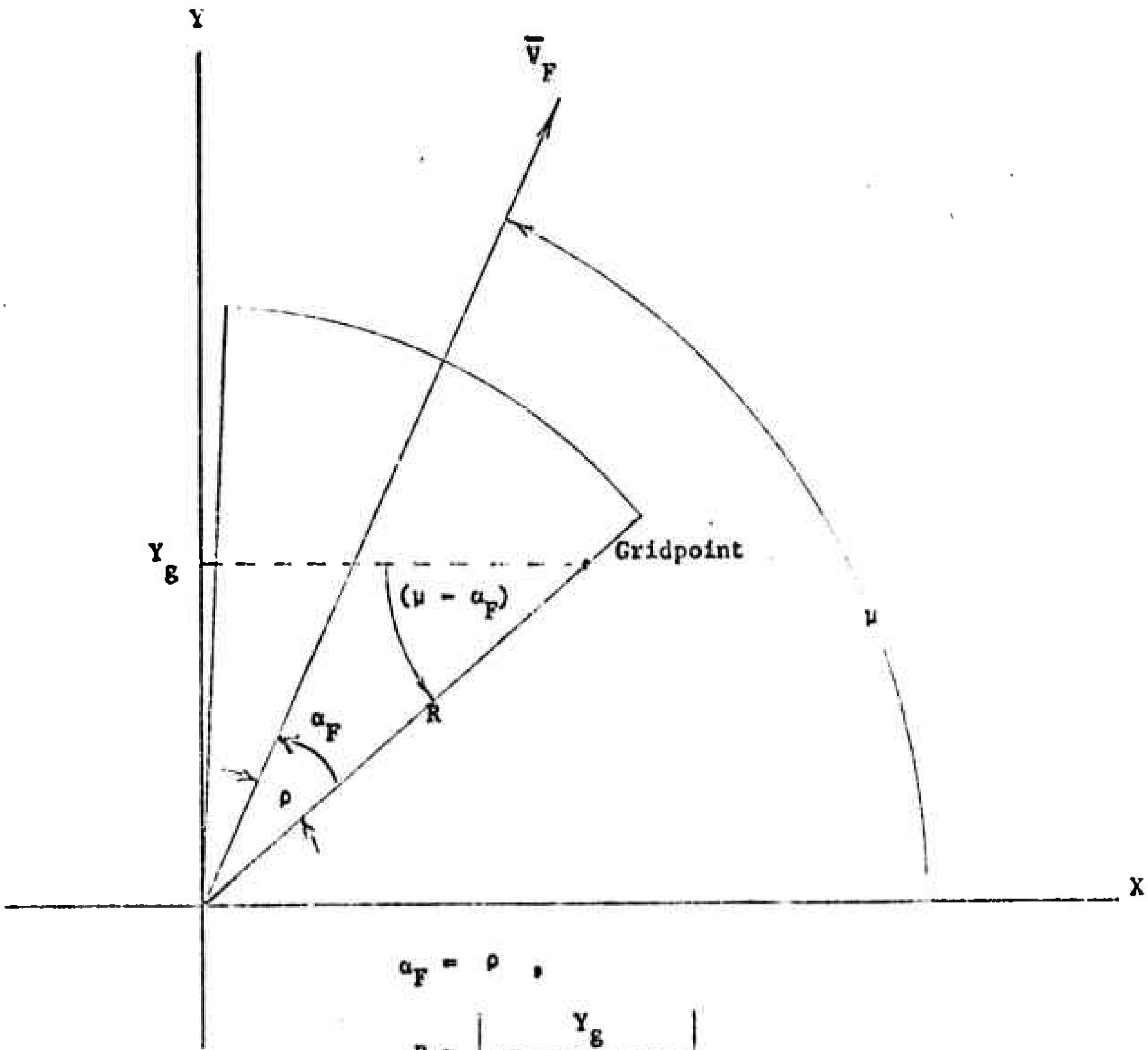
which can be derived from the construction presented in Figure 5.5-2.

Whatever the segment in which a grid-point is located, α_B is obtained from the following expression:

$$\alpha_B = \alpha_F + \varepsilon - [\mathrm{sgn}\ \varepsilon]\ \pi \quad ,$$

where [sgn ε] indicates + 1 or − 1 according to whether ε is positive or negative, respectively. The construction shown in Figure 5.5-3 is helpful in deriving this result. It should be remembered that all three angles shown are measured positive in the counterclockwise direction.

The branching logic incorporated into GRID is based upon the case presented in Figure 5.4-1 (note that B here does not relate to the location of the bomber). For this case, grid-points for initializing engagements will first be located along the arc "Y_{max} to C" ("Yes" to "$Y_g > Y_C$" question in flow chart), then on the radial section "C to F" ("No" to first two questions, "Yes" to "$Y_g > 0$"), then on the radial segment "F to B" ("No" to all questions), and finally on the arc "B to Y_{min}" ("Yes" to "$Y_g < Y_B$"). In other cases, any number of the branches may be executed. GRID PREPARATION sets the values of Y_B and Y_C so that the proper branches for a given case will be executed.



$$\alpha_F = \rho \ ,$$

$$R = \left| \frac{Y_g}{\sin(\mu - \alpha_F)} \right|$$

Figure 5.5-2 Determination of R and α_F for Gridpoint on Radial Segment of Fighter's Detection Pattern

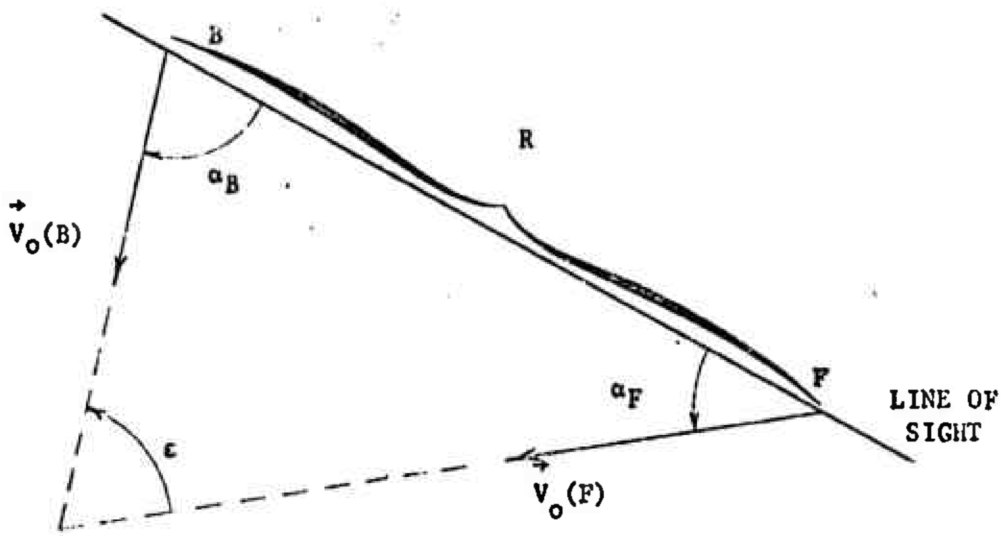


Figure 5.5-3 Determination of α_B

5.5.2 The INITIALIZE FLIGHT Routine

The INITIALIZE FLIGHT Routine (see Volume III, page 28) is executed each time a new grid-point is considered. It performs the following functions:

(1) Sets variables which describe the bomber's and fighter's flight conditions to the values which they should have at the beginning of a simulated engagement;

(2) Sets the values of control variables (e.g., ENVELOPE SW(MIS, i), ST(i), and t_{PRT}) so that branches within the various subroutines will be executed at the proper times;

(3) Sets timing variables which record the time of occurrence and duration of simulated events at their proper initial values; the clocktime is set to $t = 10^{-6}$ rather than zero, since for convenience of programming, a weapon firing should not take place at $t = 0$;

(4) Constrains the initial values of the tracking angles α_F and α_B calculated in the GRID subroutine to the interval $[-\pi, \pi]$;

(5) Calculates the initial values of the angles ϕ_F and ϕ_B within the interval $[-\pi, \pi]$;

(6) Places the bomber at the origin of the (x, y) inertial coordinate system with its velocity vector oriented in the direction of the positive y-axis ($\beta_B = \pi/2$) and places the fighter in the inertial coordinate system so that it has the proper relative position and orientation to the fighter for the given ε and grid-point;

(7) Sets the value of Δt depending on the initial range (a longer initial range allows for a longer time pulse);

(8) Sets the value of R^* , the ideal firing range. If the fighter intends to surprise the bomber, then R^* is first set to RNOW, the point at which the fighter will start firing. It will then be changed to the "true" R^* when the bomber becomes aware -- see AWARE ROUTINE.

5.5.3 NAVIGATIONAL SYSTEMS

The NAVIGATIONAL SYSTEMS Routine (see Volume III, page 29) performs the following functions:

(1) Determines the information states of an aircraft through the operation of the ACTIVE, PASSIVE, and INFORMATION Routines, and thereby determines when the bomber first becomes aware of the fighter;

(2) Decides which maneuver (linear, circular, pursuit course, linear evasion or circular evasion, defined by $ST(i) = L$, C, P, E_L, or E_C, respectively) an aircraft will execute in a given time pulse based upon the information state and the model's tactical doctrine;

(3) Sets the value of $\dot{\beta}_i$, the aircraft's turning rate;

(4) Sets the value of a_i, the aircraft's acceleration for this time pulse, within the two restraints of doctrine and the P_S function.

The values of $\dot{\beta}_i$ and a_i determine the maneuver of the aircraft. ATAC-2 focuses most sharply on (3) and (4). Section 4.6 describes the general development, and Appendices C, D, and E lay out the basis for them.

The NAVIGATIONAL SYSTEMS Routine first checks the information state (k state) of the aircraft, (see the INFO Routine, Section 5.5.3.1). The k matrix is shown in Figure 5.5-4. This k state combined with the

(k)	Information
1	None
2	Out of Time or Ammunition and No Active
3	Out of Time or Ammunition and Active, i.e., Detection Radar or Optical
4	Active
5	Active and IFF
6	Active and Passive
7	Active, Passive, and IFF
8	Passive
9	Passive and IFF
10	Lost and IFF and Inferior Turning Ability
11	Lost (other than $k = 10$)

NOTES:

States 2-11 assume awareness.

In states 4-9 information not listed is not available, e.g., $k = 4$ means no passive, no IFF.

Figure 5.5-4 Information (k) States

geometry known to the aircraft produces the m state, which defines the doctrine of acceleration and turning. The m state doctrine is listed in Figure 4.7-1. In the succeeding discussion the subscript or argument i, representing the aircraft, will frequently be dropped when inessential. The boxed symbols refer to the corresponding areas of the flow chart.

If no information exists (k state = 1), $\boxed{A}$, the linear, constant speed course is continued. In all other cases, the variable D will dictate whether to opt for evasion. If for a given k state $D = 0$ (the D's are input), then a course of evasion follows, $\boxed{B}$; otherwise the policy is to attack.

In the case of evasion, if $k = 3, 4, 5, 6,$ or 7, these information states imply knowledge of ϕ, the angle-off; then if $|\phi| < 90°$, the aircraft is aware it is behind the opponent. It will then make a decision based on the speed. If $V \geq V^*$, it will turn as hard as possible (x_2) even if this means losing speed. If $V \leq V^*$, it will limit its turning rate (x_1) so as not to lose speed. The aircraft turns in the opposite direction of the enemy $(-\operatorname{sgn} \dot{\beta}_j)$.

In all other evasion situations it will straighten out and accelerate to maximum speed $(\dot{\beta} = 0,\ V \longrightarrow V_{max})$.

If $k = 10$ or 11, see $\boxed{C}$, then the aircraft has lost all information. If $V > V^*$ it will turn as hard as possible (x_2) and decelerate. If $V \leq V^*$ it will control its turning rate to less than $g_1(V)$, specifically $g_1(V)\,\frac{V}{V^*}$, see equation (4.7-1), and accelerate. It will turn in the same direction as previously, setting the sign of $\dot{\beta}$ equal to $\operatorname{sgn}(\dot{\beta})$, the old value.

The k states 8 or 9 (passive information only), or 2 (inability to fire with no active information), see $\boxed{D}$, are treated just as k states 10 and 11; accelerate or decelerate to V^* with the turning rate restrictions mentioned above. If there are two passive sensors (PS(i) = 2) then the aircraft can tell the hemisphere of the opponent and turns into the opponent. A negative α means the opponent is on the right and so turns the aircraft right (sgn $\dot{\beta}$ = 1). The opposite is done, of course, if $\alpha > 0$. For completeness, in the unlikely event that $\alpha_1 = 0$ (pointing at the opponent and yet only have passive information), then ϕ_1 , the angle-off, distinguishes the side. If PS(i) = 1 , then it is impossible to tell which side the opponent is on, and the aircraft takes a doctrine left turn (sgn $\dot{\beta} = -1$).

In all other cases (k states 3, 4, 5, 6, 7) the aircraft has active information and should consider the pursuit doctrine. The appropriate value of the tracking angle, η , for the next pulse, must be determined, see $\boxed{E}$. The first step is to evaluate ϕ^* , the angle-off at which either pure or constant lag pursuit is adopted. If $|\phi| < \phi^*$, then η , the desired α , is set to λ , the fixed lag angle. Normally $\lambda = 0$ and pure pursuit results. If $|\phi| > \phi^*$, then the magnitude of η is set by the formula $K(\phi - \phi^*) + \lambda$. (See equation (4.5-1).) To determine the slope K the variable α_{MAX} is needed. This is the value of α at $\phi = 180°$, where the target is aimed right at the aircraft. The necessary turning rate can then be calculated directly from η and $\dot{\theta}$, the last being calculated here although recalculated in ADVANCE RELATIVE COORDINATES (Section 5.5.4). Throughout the flow, when the desired $\dot{\beta}$ cannot be achieved, the status of the aircraft reverts to circular flight.

Now we come to specialized doctrine based on geometry. If $|\phi| > \pi/2$, see $\boxed{F}$, the aircraft is in front of the target, and $S(V^*)$, the distance the aircraft closes in by the time $V = V^*$, governs the decision to accelerate or decelerate. If $R < S$, $(S = S(V^*))$, then the aircraft must slow down, and the aircraft turns as necessary. If this necessary amount exceeds the doctrine limitation $(x_1 = \dot{\beta}(g_i(V)))$, then it holds its turning rate to this limit. If $R > S$ the aircraft turns only at $\frac{V}{V^*} x_1$ (if necessary) and accelerates as specific power allows.

If $\phi \leq \pi/2$, then V_o, the ideal speed to make the range rate equal zero, is calculated. If V_o is too small, it is limited by $V_C(i)$, the minimum speed. $S(V_o)$ is needed for acceleration decisions. However $S(V_o)$ is calculated here assuming that the aircraft in fact accelerates this pulse and then decelerates. The speed V^*, used in evaluating $S(V_o)$, is the speed after this pulse if acceleration takes place.

Four cases present themselves; let $S = S(V_o)$:

Left to Right	$\boxed{G}$	$V_i > V_o$,	$R > R^* + S - \dot{R}\Delta t$,	$m_i = 2$
	$\boxed{H}$	$V_i > V_o$,	$R \leq R^* + S - \dot{R}\Delta t$,	$m_i = 1$
	$\boxed{J}$	$V_i \leq V_o$,	$R \geq R^*$,	$m_i = 3$
	$\boxed{K}$	$V_i \leq V_o$,	$R < R^*$,	$m_i = 4$

The purpose of testing R against $R^* + S - \dot{R}\Delta t$, rather than against $R^* + S$ is to insure that overshoot does not occur. Since time increases in discrete intervals, the aircraft cannot afford to accelerate if the range is such that one pulse later the range will already be less than $R^* + S$, meaning overshoot may ensue.

In $m_i = 2$, acceleration is desired, and thus $\dot{\beta}$ must be limited by $x_2 \frac{V}{V^*}$, if $V < V^*$, to make sure acceleration is permitted. In $m_i = 1$, deceleration is the choice and the aircraft turning rate need only be limited by x_2, the structural limitations. A special case arises if $a_i \Delta t + V_i < V_o$ and $R > R^*$. This means that by decelerating, the aircraft in the next pulse would be in state $m_i = 3$ which would cause acceleration. The aircraft might then oscillate between these states without ever reaching R^*. Since tha initial R^* is a range at which to start firing, this range must be reached. Thus, in this case, deceleration does not occur.

In $m_i = 3$, as noted, acceleration is the rule, and like $m_i = 2$ the turning rate is cut to guarantee some power for acceleration. State 4 ($m_i = 4$) is the most involved. The value of y^* is now set to the steady state speed in two steps, see equation (4.7-2). First, a value less than 1 for y^* means steady state speed is possible. Then, if the target's turning rate, $|\dot{\beta}_j|$, is less than the pursuer's steady state rate, the aircraft accelerates or decelerates to y^*. (In all cases $V_C < V < V_{max}$ restricts the acceleration value a_i, and at various places checks are made to insure this; sometimes only the maximum or minimum is checked, as the other is impossible in view of the input restrictions.) The remainder of state 4 follows Figure 4.7-1 in a straightforward manner.

5.5.3.1 <u>The INFORMATION Routina</u>

An aircraft will always be in one of the k states listed in Figura 5.5-4. Tha routine (see Voluma III, page 30) simply searches through to find the appropriate one.

5.5.3.2 The ACTIVE Routine

The ACTIVE Routine (see Volume III, page 31) determines whether an aircraft's detection radar or optical capability will detect the other aircraft according to the relative positions and headings of the two aircraft. The routine is activated by the INFORMATION Routine each time it is executed. Thus, the ACTIVE Routine is executed twice during each time frame of a simulated engagement, once for the fighter as the subject aircraft and once for the bomber. The outcome indicated by the ACTIVE Routine is used by tha NAVIGATIONAL SYSTEMS Routine to determine the maneuvar to be executed by an aircraft in a given time frame. If detection is indicated, the ACTIVE Routina executes the AWARE Routine. This causes the "clock" time of the current time pulse to be recorded as the tima the bomber became aware of the fighter if the detecting aircraft is the bomber and the bomber was not previously awara of the fighter.

The ACTIVE Routine assumes that the radar and optical detection patterns are completely defined by a range and a half-angla measurad with raspect to an aircraft's nose.

5.5.3.3 Tha PASSIVE Routine

The PASSIVE Routine (see Voluma III, paga 31) determines whether an aircraft will detect the presence of tha other aircraft with its passive detection equipment, according to the ralativa positions and headings of the two aircraft. It is activated by the INFORMATION Routine. The PASSIVE Routine also executes the AWARE Routine if datection is indicated so that tha time at which the bomber first becomea awara of the fighter can ba set if it has not been set previously.

The PASSIVE Routine distinguishes between passive detection equipment which has the capability to recognize whether a detected aircraft is positioned in the detecting aircraft's right or left hemisphere and that equipment which does not have this capability. If an aircraft possesses the first type of equipment, the PASSIVE Routine will cause the aircraft to circle toward the other aircraft if passive detection occurs. With the second type of equipment, a detecting aircraft will execute a tactical doctrine circling turn to its left regardless of the relative position of the detected aircraft.

Two conditions must be satisfied simultaneously for the PASSIVE Routine to indicate that passive detection has occurred. The potentially detected aircraft must be illuminating the passively detecting aircraft with its tracking radar and the detected aircraft must be positioned within the passive detection pattern of the detecting aircraft. The tracking and detection patterns are both assumed to be completely specified by a range and a half-angle. The half-angle of the tracking radar is measured from an aircraft's nose and the passive detection's half-angle from an aircraft's tail (see Figure 5.2-3). Illumination by tracking instead of detection radar is made a prerequisite for passive detection beceuse illumination by detection radar waa judged to involve an unacceptably high false alarm rate.

The preceding general discussion describes the function of the flow chart in the lower third of the PASSIVE Routine. The complications in the routine all arise from the possibility that the fighter will not turn on its tracking radar as soon as possible, precisely to prevent the bomber from picking up the signal. The bomber will always turn it on within range as the fighter is aware of the bomber anyway. At the top of the flow, if the opponent is an unaware bomber, the fighter cannot have the tracking radar on, thus the routine exits. If the opponent is an aware bomber, the routine can immediately skip down and check the geometry; similarly, if the fighter always has the tracking radar on (NUIND = 0). If NUIND ≠ 0 or 1 then the fighter does not turn on the tracking radar until IFF is established (ITEMP = 1) so that must be checked.

The variable P_1 defines a choice of doctrines. If $P_1 = 1$, then the tracking radar is turned on only when firing a weapon. Whenever $\sigma(\text{MIS}, j) \leq \Delta t$, aircraft j has just fired weapon MIS. This means that the tracking radar of aircraft j has just been turned on. The program loops through all missiles to check if any have just been fired.

If $P_1 \neq 1$, then the alternative doctrine is used: The fighter turns on its tracking radar when it estimates it is τ(input) seconds away from first entering some launch envelope ($R(\phi_j) - \dot{R}\tau$, see Section 5.5.3.4). If $\dot{R} > 0$, the aircraft are moving away from each other. In such cases, if $R > R(\phi_j)$ the tracking radar will not be turned on. The variable P_2 is initially 0. Once the bomber passively detects the fighter, P_2 becomes 1, meaning that the tracking radar is on, and stays on.

5.5.3.4 The $R(\phi_j)$ Function

This routine (see Volume III, page 32) establishes an earliest range at which the tracking radar will be turned on. It is self-explanatory in the flow chart.

5.5.3.5 The AWARE Routine

The AWARE Routine (see Volume III, page 32) is designed to record the clock time within a simulated engagement for a given ε and grid-point at which the bomber first detects the fighter with its radar, optical or passive detection capability. The variable which represents this time is t_{AWARE}. Its value is set initially to 1,000 (seconds) by the INITIALIZE FLIGHT Routine. The AWARE Routine uses this value as a key in determining whether the bomber detected the fighter before the time pulse being considered. If detection does not occur for an entire engagement, the value of t_{AWARE} remains 1,000. This value then appears in the program's printout to indicate that the bomber did not detect the fighter. When the value of t_{AWARE} is set, the variable ICAN of the opponent is set to zero to allow firings, as no further advantage is to be gained by surprise. Also, the ideal firing range, R^* is set to its smaller value, and the opponent, now that the target is aware, will come in closer. The AWARE Routine is activated by the ACTIVE and PASSIVE Routines whenever they indicate that detection has occurred. Also, since the bomber can become aware by being fired on, the CHECK WEAPONS Routine contains a duplicate of the AWARE Routine.

5.5.3.6 The $G_i(x)$ Routine

$G_i(x)$ is the structural or aerodynamic limitation on the number of g's pulled by aircraft i traveling at speed x. It is determined by interpolating across an input set of speeds. (See Volume III, page 33.)

If $B_i = 1$, then the pilot is sick, and $G_B(i) = 1$ means that the model will take this into consideration; the pilot in a sick state cannot pull more than 1.5 g's.

5.5.3.7 The $g_i(x)$ Routine

This routine (see Volume III, page 33) finds the number of g's to be pulled which just allows the aircraft to maintain speed. The routine is, in form, like the $G_i(x)$ Routine.

5.5.3.8 The $P_i(V_i, \dot{\beta}_i)$ Function

This is the specific power function referred to as P_S in Section 4 and Appendix E. The routine (see Volume III, page 34) first must evaluate $\bar{x}$, the number of total g's pulled by the aircraft. This is necessary because although the program works with $\dot{\beta}$, the input table is in terms of g's. Then the interpolation proceeds as explained.

5.5.3.9 The $\dot{\beta}(x)$ Function

This is equation (4.6-1), needed after determining the appropriate number of g's, to relate this to a turning rate. (See Volume III, page 34.)

5.5.4 ADVANCE RELATIVE COORDINATES

This routine (see Volume III, page 35) must first determine the appropriate time pulse, Δt. When the aircraft are far apart the relative geometry does not change so abruptly as when close in, and thus three values of Δt (t_D) depending on the range (RTEST) are input. Once a smaller range, and thus a smaller Δt, is used, however, the model will not revert to a larger Δt if the range again increases.

The equations for the rate of change of the variables are from Section 4.3.1. The decision as to whether to use the close in equations (Section 4.3.2) depends on the range being less than the sum of the speeds times Δt . If this is so, then conceivably using the relative equations (left hand side of flow) the range could go to zero in one pulse. The equations can also produce an ill-behaved $\dot{\theta}$, noted in 4.3.2. Thus, the right side of the flow is used instead.

The values of R , ϕ_F and α_F are determined by adding the value of each during the previous time pulse to the change in its value experienced during the given time pulse. After values of ϕ_F and α_F are determined, they are used to determine the values of ϕ_B and α_B , these being the negative supplementary angles of α_F and ϕ_F , respectively. (See Figure 5.5-5.)

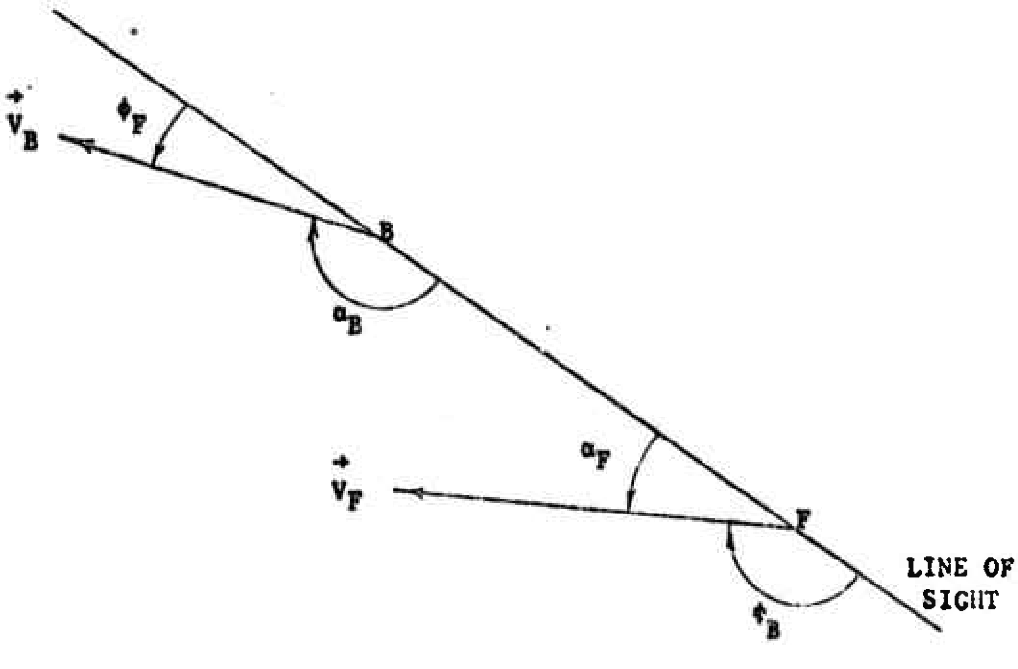


Figure 5.5-5 Determination of α_B and ϕ_B

In order to assure that the angles remain in the interval $[-\pi, \pi]$ for all subsequent operations, the PV(x) Routine (Section 5.5.10.1) is applied in all instances in which an angle is incremented in the given time pulse. The "sgn" function (Section 5.5.11.2) appears in the last two equations in order to guarantee that the angles remain in the interval.

5.5.5 TRANSFORM TO INERTIAL COORDINATES Routine

TRANSFORM TO INERTIAL COORDINATES (see Volume III, page 36) determines the positions in the inertial coordinate system which the fighter and bomber attain after executing their maneuvers in a given time pulse. The (x, y) coordinates of the aircraft are needed for output purposes to provide a graphic presentation of the flight maneuvers performed by the aircraft during an engagement. The coordinates are printed out at regular intervals as governed by the setting of t^*, the print interval, in the PRINT Routine.

TRANSFORM TO INERTIAL COORDINATES first establishes the x, y coordinates of the bomber (B). The (x, y) coordinates of the fighter (F) are then determined by adding the relative coordinates of F with respect to B to the (x, y) coordinates of B. The equations used to find the inertial coordinates of B are stated in equation (4.3.1-5) using Figure 4.3-2. In addition, the change in velocity is evaluated at the end here. This logically must be done either at the end of TRANSFORM TO INERTIAL COORDINATES or before NAVIGATIONAL SYSTEMS so that all the equations use the same velocity. In order that in the first pulse of the simulation the aircraft move at the input speeds, this incrementing is done at the end.

The procedure employed for determining x_B and y_B is executed quickly by a digital computer. This is an important consideration because this calculation is a part of the basic cycle of the ENGAGEMENT Model, a series of instructions executed many more times than any other instructions in the model. The errora involved in applying this method are not severe. If B is on a circle for a period of time, the procedure typically yields an error in B's coordinates of 1% of the radius of the circle for a quarter circle turn and virtually no error for a half-circle turn. These figures apply to a time pulse length of 0.1 seconds.

The equations used for determining F's inertial coordinates are shown in Equation (4.3.1-6) using Figure 4.3-3, where R, ϕ_F and β_B are obtained by ADVANCE RELATIVE COORDINATES. These equations simply resolve the distance R between B and F into x and y components and add them to B's inertial coordinates to obtain F's inertial coordinates.

The inertial coordinates of F can be computed directly in the same manner as those of B from equation (4.3.1-5). The set of equations (4.3.1-6) were adopted instead of computing x_F and y_F because independent computation of x_B, y_B and x_F, y_F could lead to a cumulative discrepancy in the apparent relative positions of the aircraft due to the approximate nature of the equations (4.3.1-5). Since the decision rules incorporated into the ENGAGEMENT Model, which govern the selection of aircraft maneuvers and the time at which weapons can be fired, operate on the relative position of the two aircraft, accurate presentation of relative position was considered sufficiently important to dictate the adoption of equations (4.3.1-6).

If the "close in" equations were used in the ADVANCE RELATIVE COORDINATES Routine, then it would be logically unnecessary to reevaluate the inertial coordinates here. For programming convenience it is done anyway.

5.5.6 The FIND G_i Routine

The FIND G_i Routine (see Volume III, page 36) sets the value of G_i to the number of total g's attained by aircraft i in a given time pulse. The routine is executed twice, once for each aircraft. The value of G_i determined by the FIND G_i Routine is operated upon by the CHECK WEAPONS Routine in a given time frame. In the CHECK WEAPONS Routine, a firing aircraft's G_i is compared with the maximum total g's at which each available weapon can be fired. The G_j of the target aircraft is used to select the proper firing envelopes to be used in evaluating whether the target aircraft may be fired upon for each of the firing aircraft's available weapons. The rest of the flow relates to the sickness of the pilot. The value O_i describes the lack of oxygen in the pilot's head, ranging from $O_i = 0$, meaning the pilot is all right, to $O_i = 1$, meaning the pilot is too sick to maneuver. Eight g's are too much for the pilot in any case. More than four g's causes a loss of oxygen, thus an increase in O_i, the amount of loss, depending on G_i, the number of g's. If $G_i < 1.5$, then oxygen returns, reducing O_i.

Whenever O_i becomes equal or greater than 1, the value of B_i is set to 1, meaning the pilot is sick. Initially it is zero. Once $B_i = 1$ it is not set to zero until the oxygen debt $O_i = 0$ again. S_i keeps track of the length of time the pilot is sick and so is incremented appropriately.

The variable $G_B(i)$ is an input which indicates whether to exercise the concept of pilot sickness in maneuvering. Sick time is always evaluated, but only if $G_B(i)$ is set to 1 will the pilot's turning rate be affected.

Thus, if $G_B(i) = 1$ and $B_i = 1$, the pilot is restricted by the $g_i(x)$ routine to less than 1.5 g's, and there is no need to check for that here.

5.5.7 CHECK WEAPONS Routine

The CHECK WEAPONS Routina (eee Volume III, pege 37) evaluates what weapons ere to be firad this pulse. Four conditions ere considered.

(A) The geometric rastrictions.

(B) Ammunition eupply.

(C) Firing rate.

(D) If used, the sickness of the pilot.

Over the course of e simulation for each ε and eech grid-point, the routine determines:

(1) the number of ehots fired of eech weapon typa,

(2) the time t, if any, et which each weepon type is first fired; and

(3) the total tima $t_{MIS}(i)$ within an engagement during which each waapon's firing raquirements (A) end (B) are setisfied.

If there are interruptions in satisfying (A) $t_{MIS}(i)$ is tha sum of the durations of the periods in which (A) is setisfied and ammunition is left. No initial weepon system reaction time is provided for in the routina. The first shot is fired the instant conditions are setisfectory.

The CHECK WEAPONS Routine evaluatee engagement conditions in view of various restrictions due to relative geometry and pursuit dynamics.

First, at some time before firing the target must be positioned within the IFF recognition pattern of the attacker. This requirement applies to all weapon types (except tail guns) within the firing aircraft's configuration. The IFF recognition pattern is defined by a range and half-angle measured with respect to the attacker's nose. Establishment of IFF is recorded by ITEMP(i) = 1 .

General requirements for firing are, starting from the top of the flow, that either the detection or optical sensors must perceive the target; that IFF be established, as already noted; then comes the doctrine decision to fire determined by ICAN (Section 5.5.3.5). Next the tracking radar requirements must be satisfied. The CHECK WEAPONS Routine does not terminate if some of these requirements are not met. Instead of "RETURN" the flow goes to 1 which says to consider the last missile, $n_m(i)$, the tail guns. These do not need certain of the usual requirements for firing.

If the general requirements are satisfied, each missile is considered separately at the box subsequent to 2 . If the missile cannot be fired, the flow goes to 3 where the next missile is considered. Two limitations are checked which may be different for each missile: The tracking angle and the g's of the attacker. The case of tail guns is checked separately (off to the left), since $\pi - \alpha$ is its "tracking" angle, rather than α . If the amount of ammunition expended, k(MIS, i) is less than the amount originally on board, N(MIS, i) , then there is ammunition available and the weapon is checked against its envelope. This final set of requirements includes maximum and minimum ranges for the firing of each type of weapon, for which the R_{MIS} , R'_{MIS} Routine is used. These ranges are expressed as a function of the target's lateral g's, the average of the two aircraft's speeds, and the attacker's angle off the target's tail. In addition, missiles one and two may be required to fire outside a minimum range RFLOOR.

If the range between the firing and target aircraft falls between R'_{MIS} and R_{MIS}, and the pilot is not sick (see next paragraph), the weapon may be fired subject to the maximum firing rate. If these range requirements are met, the quantities $t_{MIS}(i)$ and $\sigma(MIS, i)$ are incremented. The variable $t_{MIS}(i)$ is the length of time in the envelope of weapon type MIS, and $\sigma(MIS, i)$ is the time since the last firing of MIS. A check is then performed to determine if this weapon has been fired previously. The latter check is accomplished with the aid of the ENVELOPE SW(MIS, i) switch. At the beginning of an engagement, this switch is set to the value OFF by the INITIALIZE FLIGHT Routine. It remains at this setting until the first of this weapon type is fired. When this occurs, the switch is set to ON, and the clock time and quantities that describe flight conditions for the given time pulse are stored. These quantities are the range between the two aircraft, the attacker's tracking angle α_i and the angle-off ϕ_i. The values of these quantities at this time are identified as RESULT(MIS, i). Setting the value of ENVELOPE SW(MIS, i) to ON prevents RESULT(MIS, i) from being replaced in storage by a new set of values in some subsequent time pulse.

If $S_F(i)$ is 1, then the oxygen debt concept is to be exercised. If $O_i = 0$ there is no oxygen loss, and $O_i = 1$ means the pilot is sick. As mentioned in Section 5.5.3.6, a variable $G_B(i)$ determines whether the pilot's sickness affects his turning. Similarly, $S_F(i) = 1$ means that the

O_i affects his weapon firing rate. If $O_i = 0$, then firing should proceed at the input firing interval, τ. If $O_i = 1$, then firing should not take place, or, equivalently, the firing interval is infinite. The exponential function in the flow forms a continuity assumption about this interval between firings as the oxygen debt increases from 0 to 1. The logic requires that this S_F and O_i be checked twice (once on the middle of the page, once near the bottom). This is because the oxygen debt can be a requisite for first setting the ENVELOPE SWITCH to ON; thus it must be checked independently there.

If the envelope switch is turned on this pulse, then a duplicate of the AWARE Routine (Section 5.5.3.5) is exercised and from there the procedure is the same whether or not the envelope is first being entered. If this is the first firing of this weapon, then there is no delay and the aircraft fires immediately, the oxygen debt permitting. The τ interval question can then be ignored. Firing is indicated by the box in which $k(MIS, i)$, is incremented. In addition, the time of the firing and the characteristics of ℓ are recorded. This variable ℓ keeps track of the last weapon fired, $MI(\ell)$, which aircraft fired it, $IA(\ell)$, and the time of firing, $T(\ell)$.

The cycle is repeated for each weapon in an aircraft's configuration for each time pulse.

5.5.7.1 The R_{MIS}, R'_{MIS} Routine

The R_{MIS}, R'_{MIS} Routine (see Volume III, page 38) is an interpolation procedure. It determines for a given weapon, the minimum and maximum range constraints. These depend on the engagement conditions at the time, as specified by the aircraft velocities, attacker's angle-off, target's total g's and direction of turn. The routine is used exclusively by the CHECK WEAPONS Routine. Data upon which the R_{MIS} , R'_{MIS} Routine operates are input in tabular form. The tables describe firing envelopes within which the firing of a weapon can occur. The tables are for given values of aircraft speeds (assumed to be the averages of the attacker and target speeds) and the target's lateral g's. A diagram of a firing envelope is shown in Figure 4.4-1.

The R_{MIS} , R'_{MIS} Routine linearly interpolates along both the average velocity and the target's lateral g's. There are six firing tables for each weapon type, a pair of tables for each of three values of the target's lateral g's. Each pair consists of a maximum range table denoted $R_{FK}(V_a , o_b)$ and a minimum range table, $R'_{FK}(V_a , o_b)$. The entries for a given V_a in a pair of tables describe a firing envelope such as that illustrated in Figure 4.4-1. In this diagram, j is the target and i is the attacker. The target is traveling to the right. The firing tables apply to a target aircraft executing a right turn ($\dot{\beta}_j > 0$). The firing envelopes which apply to a left turn are obtained by reflecting the right turn envelopes about the target aircraft's longitudinal axis. This is accomplished in the CHECK WEAPONS Routine through the quantity o_i which is equal to ϕ_i if the target aircraft is making a right turn and is equal to $- \phi_i$ if the target aircraft is making a left turn.

After determining the o_b of the table closest to the o_i of a given time pulse, the R_{MIS} , R'_{MIS} Routine determines the tabular values of g's bracketing the target's present g level. The target's g's are given in terms of total g's at this point in the program, and a conversion to lateral g's is effected. If the number of lateral g's pulled by the target is beyond the extremes of the tabular g's, the routine takes the firing ranges for the last two extreme tabular values and extrapolates.

Having determined the upper and lower tabular g's, the routine selects the two range tables corresponding to each of these tabular g's. From these four tables linear interpolation is performed on velocity to arrive at four range values, values of minimum and maximum firing ranges for the two selected tabular g values. These quantities are denoted Z_K , Z_K' , Z_L and Z_L' in the flow chart of the R_{MIS} , R_{MIS}' Routine. Using these quantities, the routine then interpolates with respect to lateral g's to obtain the quantities R_{MIS} and R_{MIS}' .

5.5.8 <u>The PRINT Routine</u>

The principal purpose of the PRINT Routine (see Volume III, page 39) is to provide data for plotting each aircraft's position in inertial coordinates as a function of time. The input t^* defines the amount of time between successive printings of the (x , y) coordinates of each aircraft. Of course, t^* should be an integral multiple of the time increment Δt .

Other outputs printed include:

t - the time

R - the range

and for each aircraft:

k_i state (see Figure 5.5-4)

m_i state (see Figure 4.7-2)

ST(i) status (see Section 5.5.3)

V_i - the apsed

$\dot{\beta}_i$ - the turning rate

α_i - the tracking angle

ϕ_i - the angle-off

5.5.9 The OVER Routine

The OVER Routine (aee Volume III, page 39) determines whether or not the conditions which terminats an engagement have been mst. If these conditions have been met, OVER will cauae a branching to the RESULTS Routine which stores pertinent data. If the conditions have not been met, control is returnsd to COMBAT for another iteration.

An engagement is terminated by any of the following conditions:

(1) Neither combatant has information about the other and the current value of t is greater than the input t_{MIN},

(2) the current value of t is greatsr than the input t_{MAX}, or

(3) the range R is less than the input R_{MIN}.

If an engagement is terminated, the valus of time t is aaved as t_{LAST}, the duration of the engagement.

5.5.10 The RESULTS Routine

The purpose of the RESULTS Routine (see Volume III, page 40) is to store information which describea the final resulta of an engagement for eventual printout and use in the DATA PROCESSING Model of ATAC-2. The RESULTS Routine operatea once for each grid-point. A check is first made to determine the awareness of the b...ber. The information stored includes the initial conditions of an engagement, the results and an identification of the computer run. Specifically, the items stored are:

(1) ID -- the computer run's identification which includes the run number, altitude of engagement, aircraft identifications and weapon identifications;

(2) ε -- the initial crossing angle of the bomber's and fighter's flight paths;

(3) GRIDPOINT -- the number of the grid-point for the given ε ;

(4) t_{AWARE} -- the time at which the bomber became aware of the fighter. It is set to 1,000 if the bomber never became aware.

(5) t_{LAST} -- the duration of the engagement;

(6) RESULT (MIS, i) -- the values of time (t), range between aircraft (R), i's tracking angle (α_i), and the angle-off (ϕ_i) at the first firing of each type weapon;

(7) $t_{MIS}(i)$ -- the cumulative time during which each weapon type could have been fired if there were no restriction on firing rate and ammunition limitation;

(8) F -- a factor, used in the DATA PROCESSING Model, which is a function of the fighter's active detection pattern and the relative velocity of the two aircraft;

(9) The history of weapon firings shown by: ℓ , MI(ℓ) , IA(ℓ) , T(ℓ);

(10) S_i -- the length of time each pilot was sick during the engagement.

5.5.11 General Purpose Routines

5.5.11.1 The PV(x) Function

The PV(x) (principal value) Function (see Volume III, page 40) determines an angle that is both equivalent to its argument and in the interval $[-\pi, \pi]$. It insures that the principal value of an angle will enter into subsequent calculations when an angle is obtained by the addition of two other angles. The function is defined to be:

$$PV(x) = \begin{cases} x \; , & \text{if } |x| \leq \pi \; , \\ x - 2\pi \; , & \text{if } x > \pi \; , \\ x + 2\pi \; , & \text{if } x < -\pi \; . \end{cases}$$

It is sufficient to subtract or add 2π to obtain an angle in the interval $[-\pi, \pi]$ equivalent to the argument, because each of the angles added to obtain an argument is always less than π in absolute value, and no more than two angles are ever included in a sum. That the angles will be restricted to the interval $[-\pi, \pi]$ is assured by applying the PV(x) Routine whenever two angles are added or an angle's value is incremented.

5.5.11.2 The SGN(x) Function

The purpose of the SGN(x) Function (see Volume III, page 41) is to give the sign of its argument. It is used in various expressions for the sake of brevity. The function is defined as

$$\text{sgn}(x) = \begin{cases} 1 \; , & \text{if } x \geq 0 \; , \\ -1 \; , & \text{if } x < 0 \; . \end{cases}$$

5.5.11.3 The Q(x) Function

The purpose of the $Q(x)$ Function (see Volume III, page 41) determines the quadrant of its argument. It is used by the GRID PREPARATION Routine to determine the quadrant of θ_B and θ_C where the values of the angles are confined to the range $[-\pi, \pi]$. $Q(x)$ is defined to be

$$Q(x) = \begin{cases} 1 , & \text{if } \pi/2 \geq x \geq 0 , \\ 2 , & \text{if } \pi \geq x > \pi/2 , \\ 3 , & \text{if } -\pi \leq x < -\pi/2 , \\ 4 , & \text{if } -\pi/2 \leq x < 0 . \end{cases}$$

SECTION 6

DATA PROCESSING MODEL DISCUSSION

6.1 Introduction

The purpose of the DATA PROCESSING Model (DPM) is to compute various conditional probabilities of kill of the combatants in the ENGAGEMENT Model (EM). Being a separate program, the DPM utilizes the stored results of the EM along with its own inputs to compute these probabilities. The basic results of the EM are the sequences of firings of each combatant for each engagement; the basic inputs of the DPM are the individual single shot probabilities of kill of the weapons fired by the combatants. One can perform parametric variations on these weapon probabilities or one can vary the weapon loads of each combatant in the DPM while utilizing the same results of the EM, thus making re-runs of the EM unnecessary.

An encounter will mean a random engagement. The objective of the DPM is to render probabilities of kill for an encounter, independent of initial conditions, from the individual results of each engagement. To obtain probabilities associated with an encounter, the probabilities for each engagement are averaged, so to speak, over all grid-points to obtain probabilitiea as a function of ε, the initial relative heading angle. These probabilities are in turn averaged over the various values of ε to obtain the encounter probabilities.

Section 6.2 delineates the method of obtaining the engagement probabilities, while Section 6.3 contains the derivation of the encounter probabilities from the engagement probabilities. The expected number of enemy aircraft killed over an aircraft's lifetime is derived in Section 6.4.

Finally, consideration is given to the computational aspects of evaluating the above probabilities in Section 6.5.

6.2 Engagement Probabilities of Kill

In this section the concern shall be with the calculation of the probability of kill of the combatants of the EM based on the information provided by the EM and the separate inputs of the DPM. For each engagement the EM provides a list of times of all weapon firings. The single shot probabilities of kill of various weapons are used by the DPM only.

Throughout this section the terms "killed" and "alive," or "dead" and "survived" are used to describe the condition of an aircraft at some point in time. No partial damage is considered in this model; a weapon either totally destroys its target or leaves it unharmed. In a sense, the DPM restricts the firings of weapons. A weapon is considered fired in the DPM only if its target is alive at the time of firing. The weapon takes an amount of time t_f ("time of flight") to get to target whereupon it either misses or destroys the target.

Consider an engagement in which the EM indicated various weapons were fired by each combatant. Let z be the total number of weapons fired by the two aircraft during the engagement. Also let $T(\ell)$ $(\ell = 1, 2, \ldots, z)$ be the time at which the ℓ^{th} weapon was fired. The times are in ascending order; the ℓ^{th} weapon was fired before or at the same time as the $(\ell + 1)^{st}$, hence, $T(\ell) \leq T(\ell + 1)$, $\ell = 1, 2, \ldots, z - 1$.

Associated with time $T(\ell)$ is the aircraft that fired the ℓ^{th} weapon, $IA(\ell)$, and the weapon type, $MI(\ell)$. The aircraft, $IA(\ell)$, takes values of i and j (or F and B). The missile number, $MI(\ell)$, is the type number of the weapon fired; thare may have been many firings of this weapon type by an aircraft during an engagement. Let the single shot probability of kill associated with the ℓ^{th} firing be $P_k'(MI(\ell), IA(\ell))$. Thesa probabilities are tha basic inputs to the DPM.

A weapon fired at time $T(\ell)$ hits its target at time $T(\ell) + t_f$. Supposa then that the probability that aircraft j is killed during the engagement is desired. Let $P(\ell)$ ba tha probability that aircraft j is dead at time $T(\ell) + t_f + 0$[1], just after the possible hit of the ℓ^{th} weapon. (It is evident that the time dependent probability of kill may be uniquely described by $P(\ell)$ since an aircraft may be killed only at times $T(\ell) + t_f + 0$.) Let $p(\ell)$ be the probability that the ℓ^{th} weapon alone -- and no other weapon -- kills aircraft j. Then the guiding relationship throughout this development is that

$$P(\ell) = \sum_{n=1}^{\ell} p(n) \quad . \qquad (6.2\text{-}1)$$

This is interpreted as the total probability that aircraft j is dead at time $T(\ell) + t_f + 0$ is equal to the sum of the effects of all weapons that could have hit (and therefore killed) aircraft j -- all those that hit on or before $T(\ell) + t_f$. From (6.2-1) it is evident that

$$P(\ell) = P(\ell - 1) + p(\ell) \ , \ \ell = 2, 3, \ldots, z \quad . \qquad (6.2\text{-}2)$$

---

1) The notation $+ 0$ and $- 0$ is needed to describe whether the point in time $T(\ell) + t_f$ is or is not included. Here the time of the hit must be included.

Then having derived $P(1)$ and $p(\ell)$, the probabilities $P(\ell)$ may be computed seriatim for $\ell = 2, 3, \ldots, z$. The major portion of the balance of this section is devoted to deriving $p(\ell)$. Equation (6.2-2) is used in the DPM. It is a general equation with ℓ as an index of the time order of the interlaced firings by the two aircraft. Two basic cases arise; the ℓ^{th} weapon could have been fired by either aircraft in the EM. For the trivial case that aircraft j fired the ℓ^{th} weapon at time $T(\ell)$ it is clear that this will not affect the survival of j at time $T(\ell) + t_f + 0$. Hence

$$p(\ell) = 0 \ , \ \text{if} \ IA(\ell) = j \ . \qquad (6.2\text{-}3)$$

Now consider the other case; the EM indicates that aircraft i fired the ℓ^{th} weapon at aircraft j, that $IA(\ell) = i$. In this case $p(\ell)$ will, in general, not be zero. The event that the ℓ^{th} weapon kills (hits) aircraft j will now be described in terms of an equivalent set of events, the probabilities of which are more easily obtained.

For the ℓ^{th} weapon to kill aircraft j it is necessary and sufficient for the following three events to take place:

A) Aircraft j is alive just before the possible hit of the ℓ^{th} weapon,

B) Aircraft i is alive to fire the ℓ^{th} weapon,

C) The ℓ^{th} weapon, in fact, hits aircraft j and thereby kills it.

Further, the single shot probability of kill of the ℓ^{th} weapon, $P_k'(MI(\ell), i)$, is by definition the probability that the target is killed by this weapon, given that the weapon is fired and the target is alive to be hit by it. Thus,

$$P_k'(MI(\ell), i) = \Pr\left\{ \ell^{th} \text{ weapon kills } j \,|\, j \text{ alive at } T(\ell) + t_f - 0 \text{ , } i \text{ alive at } T(\ell) \right\}$$

$$= \Pr\left\{ C \,|\, A \text{ , } B \right\} \quad .$$

But

$$p(\ell) = \Pr\left\{ C, A, B \right\} \quad ,$$

thus

$$p(\ell) = P_k'(MI(\ell), i) \Pr\left\{ A, B \right\} \quad , \qquad (6.2\text{-}4)$$

This means the probability that aircraft j survives to $T(\ell) + t_f - 0$ and i survives to $T(\ell)$ is required. This joint event is further decomposed. Consider the events

D) Aircraft j survives from $T(\ell)$ to $T(\ell) + t_f - 0$,

and

F) Aircraft j survives to $T(\ell)$.

Event A is equal to the joint event D and F. The probability of the combined events of A and B may be obtained as follows:

$$\Pr\left\{ A, B \right\} = \Pr\left\{ F, D, B \right\}$$

$$= \Pr\left\{ D \,|\, F, B \right\} \Pr\left\{ F, B \right\}$$

$$= \Pr\left\{ j \text{ survives from } T(\ell) \text{ to } T(\ell) + t_f - 0 \,\middle|\, i \text{ and } j \text{ both survive to } T(\ell) \right\} \Pr\left\{ i \text{ and } j \text{ both survive to } T(\ell) \right\} . \qquad (6.2\text{-}5)$$

These probabilities, being the final decomposition, will now be derived.

Consider the probability, $Pr\left\{F, B\right\}$, of the event that both aircraft are alive at $T(\ell)$. This event obtains if and only if all weapons that could hit either aircraft before $T(\ell)$ in fact do not. Let $q(\ell)$ be this probability,

$$q(\ell) \equiv Pr\left\{F, B\right\}$$

$$= \prod_{n=1}^{k} [1 - P_k'(MI(n), IA(n))] \quad , \qquad (6.2\text{-}6)$$

where k is the last weapon (if any) that could hit before $T(\ell)$; k is the largest integer for which

$$T(k) + t_f < T(\ell) \quad . \qquad (6.2\text{-}7)$$

If there is no $k \geq 1$ which satisfies (6.2-7), $Pr\left\{F, B\right\} = 1$, by definition.

The second factor on the right side of (6.2-5) has, therefore, been obtained as (6.2-6). The first factor, the probability that j survives from $T(\ell)$ to $T(\ell) + t_f - 0$, given both survived to $T(\ell)$, is now required. This probability is $Pr\left\{D|F, B\right\}$ and will also be termed $y(\ell)$ hereafter. By the above argument, aircraft j is killed during the time interval $T(\ell)$ to $T(\ell) + t_f - 0$, given both aircraft survived to $T(\ell)$, if and only if all weapons that could hit aircraft j in this time interval in fact do not hit. Thus $y(\ell)$ is the product of the survival probabilities over the set of weapons that could hit in this time interval. The k^{th} weapon, defined in (6.2-7), is not in this set; by definition the k^{th}

weapon hits before $T(\ell)$. But it is the last weapon to hit (possibly) before $T(\ell)$; therefore, the $(k+1)^{st}$ weapon hits (possibly) after $T(\ell)$. On the other end of the interval, the $(\ell - 1)^{st}$ weapon is the last to hit before $T(\ell) + t_f - 0$. Therefore, the relevant set of weapons, indexed by n , that could hit in the interval $T(\ell)$ to $T(\ell) + t_f - 0$, consists of $n = k+1, k+2, \ldots, \ell - 1$. It follows that

$$y(\ell) = \prod_{n=k+1}^{\ell-1} [1 - P_k'(MI(n), i)] \quad , \qquad (6.2\text{-}8)$$

where it is understood that the product is taken over only those weapons n for which $IA(n) = i$. A singularity occurs here when the $(k+1)^{st}$ weapon is the ℓ^{th} . No weapons can hit in the interval and $y(\ell) = 1$, if $k + 1 = \ell$.

Combining the above, $p(\ell)$ may be obtained. From (6.2-4) and (6.2-5), if $IA(\ell) = i$,

$$\begin{aligned} p(\ell) &= P_k'(MI(\ell), i) \;\; \Pr\{D|F, B\} \;\; \Pr\{F, B\} \\ &= P_k'(MI(\ell), i) \;\; y(\ell) \;\; q(\ell) \quad . \end{aligned}$$

Thus in (6.2-2) $P(\ell)$ is obtained for $\ell = 2, 3, \ldots, z$.

It remains to determine $P(\ell)$ for $\ell = 1$. Define $P(0)$ to be the probability that aircraft j is dead before the first weapon hits. Then $P(0) = 0$. Using (6.2-2) it follows that

$$P(1) = p(1) = \begin{cases} P_k'(MI(1), i) \;, & \text{if } IA(1) = i \\ 0 \;, & \text{otherwise} \end{cases} \quad . \qquad (6.2\text{-}9)$$

One more special case remains. If k does not exist, $y(\ell)$ is not defined by equation (6.2-8). This corresponds to the event that the ℓ^{th} weapon is fired before the first weapon hits the target. Then

$$T(1) + t_f > T(\ell) \quad . \qquad (6.2\text{-}10)$$

Then clearly both aircraft are alive with probability 1 at $T(\ell)$. So $y(\ell)$ is just $1 - P(\ell - 1)$, the probability that aircraft j survived the first $\ell - 1$ firings. Hence, for any ℓ for which $IA(\ell) = i$ and (6.2-10) hold, $P(\ell)$ is given by

$$P(\ell) = P(\ell - 1) + [1 - P(\ell - 1)] \; P_k'(MI(\ell), i). \quad (6.2\text{-}11)$$

Thus, $P(\ell)$, the probability that aircraft j is killed at time $T(\ell) + t_f + 0$, may be computed for all values of ℓ . By extending this process through to the last weapon fired, within the combat time limitations of the aircraft, the probabilities of kill for the engagement is found. Of course, by switching the aircraft to which i and j are assigned the probabilities are computed for the other aircraft by repeating the above process. These probabilities are called $P_j^m(\epsilon, n)$ as explained in the next section.

6.3 Encounter Probabilities

This section presents the derivations of the kill probabilities for a random engagement — an encounter. The process by which these encounter probabilities are obtained is basically an averaging of engagement kill probabilities with appropriate consideration of the conditional events.

In the previous section it was shown how the individual engagement probabilities of kill were obtained. The section defines an algorithm by which these probabilities are computed. Now, each engagement has associated with it an ε , the initial relative heading and an n , the grid-point. Together they define the initial conditions of an engagement. Heretofore this notation has been suppressed when discussing $P(\ell)$. However, it is required henceforth. Each individual engagement probability will be indexed by ε , and n . Also, further capabilities require special notation. It is interesting to compute the individual engagement probabilities of kill of each aircraft with its lethality suppressed. This gives a measure of the effect of that aircraft's weapons on its own survivability. The condition of the suppression of an aircraft's kill power (the single shot probabilities of kill) is indicated by the value of an index m in the following discussion. If m has a value of 1, then aircraft j's single shot probabilities of kill of all its weapons are taken to be zero in what follows. Hence $P_j^m(\varepsilon, n)$ for $m = 1, 2$ and $j = F$ and B are the probabilities that aircraft j is killed in the engagement identified by ε and n where the value of m indicates that j's weapons were or were not suppressed, respectively. These probabilities are obtained from the above (Section 6.2) algorithm by setting the proper values of i and j and m . If m is set to 1, then $P_k'(MIS, j) \equiv 0$ for all MIS. If $m = 2$, $P_k'(MIS, j)$ is set to the input single shot probabilities of kill, $P_k'(MIS, j)$.

One last introductory consideration must be pointed out. All the above probabilities apply to engagements in which the bomber became aware. The other case, in which the bomber was totally unaware for some engagement, requires special consideration. For this case $P_B^\circ(\varepsilon, n)$ is defined as the probability that the bomber is killed for the engagement defined by ε and n .

Further, this probability is defined only for those engagements in which the bomber was unaware. In such engagements the probabilities $P_j^m(\epsilon, n)$ $(m = 1, 2)$ are not defined. There is no analogous probability for the fighter since it is always aware.

To obtain the kill probabilities as a function of ϵ only, independent of the grid-points for that ϵ, let $P_j^m(\epsilon)$ be the probability that aircraft j is killed, for an initial relative heading angle of ϵ, given that the bomber was aware and given the previously described condition indicated by the value of m. Likewise, $P_B^*(\epsilon)$ is the probability that the bomber is killed, given it is unaware for the value ϵ. Also let $N_u(\epsilon)$ be the number of grid-points, out of the N possible, in which the bomber was unaware for the value ϵ. The distribution across grid-points is assumed to be uniform, i.e., one grid-point is as likely as another. Then

$$P_j^m(\epsilon) = \frac{\sum_{n=1}^{N} P_j^m(\epsilon, n)}{N - N_u(\epsilon)} , \quad \text{if} \quad N_u(\epsilon) < N , \qquad (6.3\text{-}1)$$

and

$$P_B^*(\epsilon) = \frac{\sum_{n=1}^{N} P_B^*(\epsilon, n)}{N_u(\epsilon)} , \quad \text{if} \quad N_u(\epsilon) \neq 0 , \qquad (6.3\text{-}2)$$

where the summations in each instance are taken over the cases which apply; there will be $N - N_u(\epsilon)$ terms in the summation to obtain $P_j^m(\epsilon)$ and $N_u(\epsilon)$ terms in the summation to obtain $P_B^*(\epsilon)$. There may have been no case in which the bomber was unaware, however. In such a case $P_B^*(\epsilon)$ is undefined. Likewise $P_j^m(\epsilon)$ is undefined if $N_u(\epsilon) = N$. The values are arbitrarily set to 2 in such cases for identification; the values are immaterial since the kill probabilities will be combined with probabilities of awareness and unawareness, which will be zero in the undefined cases.

To complete the probabilities at the ε level two more quantities are defined. Let $P_D(\varepsilon)$ be the probability that the fighter detects the bomber at an initial heading angle of ε. This probability is derived separately, in Appendix A. Also, $P_u(\varepsilon)$ is the probability that the bomber is unaware given the fighter detects the bomber for an initial relative heading angle of ε and is computed as

$$P_u(\varepsilon) = N_u(\varepsilon)/N \quad . \qquad (6.3\text{-}3)$$

This completes the necessary probabilities as a function of ε, and leaves the averaging across values of ε to obtain the relevant probabilities of kill for an encounter — independent of the initial conditions.

First considerations are to the method of averaging. The results for each value of ε are intended to represent engagements that arise from detection within an arc portion of angular width $\Delta\varepsilon$ about a circle or semi-circle (see Figure 6.3-1). The first and last values of ε, ε_1 and ε_2 respectively, have special meaning. ε_1 is set to 0° for the symmetric semi-circle or - 180° for the asymmetric case, while ε_2 is always set to 180°. The arc portions represented by ε_1 and ε_2 are each of width $\Delta\varepsilon/2$ only. Thus, in the following averaging process the results for the ε_1 and ε_2 values are discounted by 1/2 each, resulting in a total weighting by all ε of $N_\varepsilon - 1$.

Since the event of detection is a basic condition on almost all of the probabilities to follow, the probability of detection for an encounter, P_D, is obtained first. From the above consideration,

$$P_D = \frac{\sum\limits_{\varepsilon \neq \varepsilon_1 \varepsilon_2} P_D(\varepsilon) + .5 \sum\limits_{\varepsilon_1, \varepsilon_2} P_D(\varepsilon)}{N_\varepsilon - 1} \quad . \qquad (6.3\text{-}4)$$

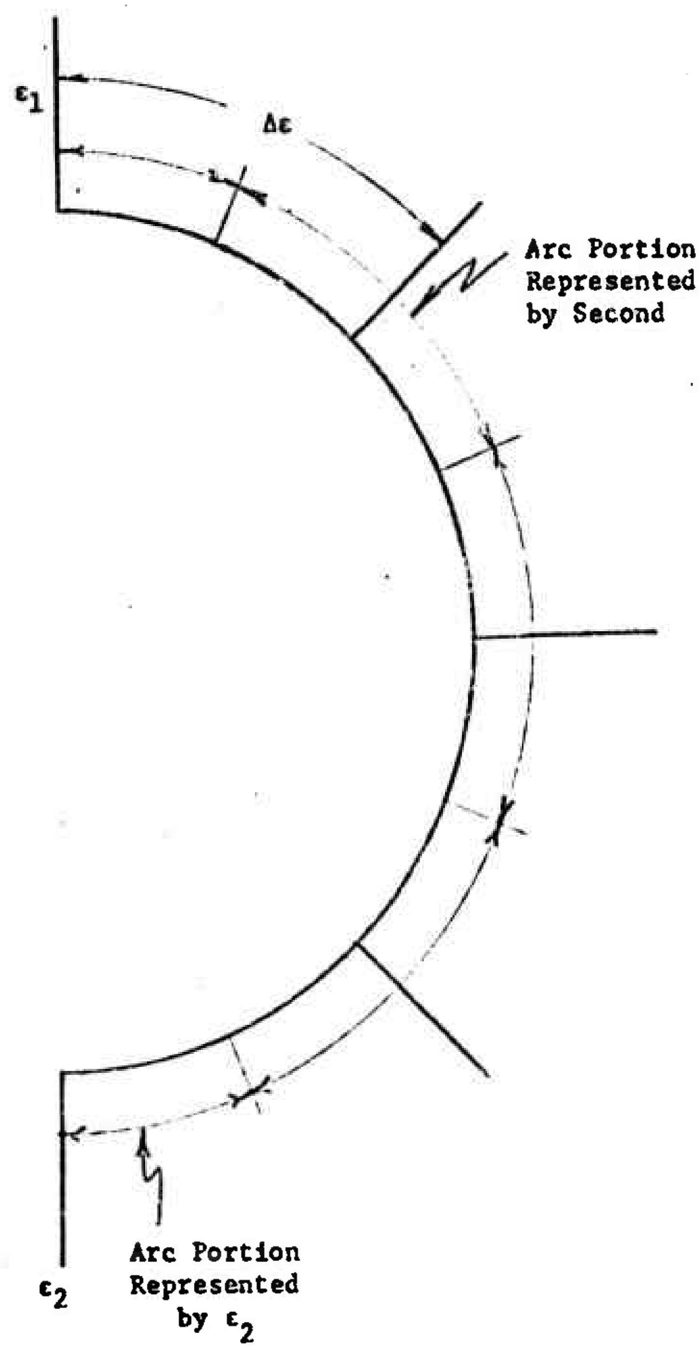


Figure 6.3-1 Arc Portions Pepresented by ϵ's

Then $P_D(\varepsilon) \cdot P_u(\varepsilon)$ is the probability that the bomber is unaware and the fighter detects the bomber for the value of ε. Summing as before over values of ε gives

$$P_u = \frac{\sum_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_D(\varepsilon)\, P_u(\varepsilon) + .5 \sum_{\varepsilon_1, \varepsilon_2} P_D(\varepsilon)\, P_u(\varepsilon)}{N_\varepsilon - 1}, \quad (6.3\text{-}5)$$

the probability that the bomber is unaware and is detected for an encounter. And the probability that the bomber is unaware, given that it is detected for an encounter is given by

$$P_{u|D} = \frac{P_u}{P_D}. \quad (6.3\text{-}6)$$

For the encounter probabilities of kill let PK_B° be the probability that the bomber is killed, given that it is unaware and detected by the fighter. Then

$$PK_B^\circ = \frac{\sum_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_u(\varepsilon) P_D(\varepsilon) P_B^\circ(\varepsilon) + .5 \sum_{\varepsilon_1, \varepsilon_2} P_u(\varepsilon) P_D(\varepsilon) P_B^\circ(\varepsilon)}{P_u(N_\varepsilon - 1)}. \quad (6.3\text{-}7)$$

Similarly, let PK_j^1 be the probability that aircraft j is killed, given that the bomber is aware, and the fighter detects the bomber, and aircraft j does not fire; let PK_j^2 be the same probability without the condition of no firings by aircraft j. Then

$$PK_j^m = \frac{\sum_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_D(\varepsilon)[1 - P_u(\varepsilon)]\, P_j^m(\varepsilon) + .5 \sum_{\varepsilon_1, \varepsilon_2} P_D(\varepsilon)[1 - P_u(\varepsilon)]\, P_j^m(\varepsilon)}{(P_D - P_u)\,(N_\varepsilon - 1)},$$

(6.3-8)

for $m = 1$ and 2 and $j = F$ and B . Notice that $P_D - P_u = P_D(1 - P_{u|D})$ is the probability that the bomber is detected and aware for an encounter.

Further encounter probabilities are of interest. Let PKF and PKB be the unconditional probabilities of kill of the fighter and bomber, respectively. Noticing that the fighter may not be killed without the bomber being aware, it follows that

$$PKF = PK_F^2 \, P_D(1 - P_{u|D}) \quad , \qquad (6.3\text{-}9)$$

$$PKB = PK_B^2 \, P_D(1 - P_{u|D}) + PK_B^* \, P_D \, P_{u|D} \quad . \qquad (6.3\text{-}10)$$

Also, let P_{SF} and P_{SB} be the unconditional probabilities that the fighter and bomber, respectively, survive. Then

$$P_{SF} = 1 - PKF \quad , \qquad (6.3\text{-}11)$$

and

$$P_{SB} = 1 - PKB \quad . \qquad (6.3\text{-}12)$$

These probabilities are of interest in computing the expectations EKB and EKF (see Section 6.5).

Let $P_e(\epsilon, n)$, $P_e(\epsilon)$ and PKE be the probabilities of kill and detection of the bomber on or before it is aware (mnemonics are e, E for early), for

1) the engagement,
2) the value of ϵ and
3) encounter, respectively.

The probability $P_e(\varepsilon, n)$ is obtained from $P(\ell)$ for the largest value of ℓ such that

$$T(\ell) \leq t_{AWARE}$$

for the applicable engagement. The other probabilities are

$$P_e(\varepsilon) = (1/N) \sum_{n=1}^{N} P_e(\varepsilon, n) \quad , \qquad (6.3\text{-}13)$$

$$PKE = \frac{\displaystyle\sum_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_D(\varepsilon) P_e(\varepsilon) + .5 \sum_{\varepsilon_1, \varepsilon_2} P_D(\varepsilon) P_e(\varepsilon)}{N_\varepsilon - 1} \quad . \qquad (6.3\text{-}14)$$

Let $S_i(\varepsilon, n)$, $S_i(\varepsilon)$ and S_i be the expected fraction of combat time that the pilot of aircraft i spends in a condition such that $0_i \geq 1$, a "sick condition," for

1) an engagement,
2) the value of ε , and
3) an encounter, respectively.

The value $S_i(\varepsilon, n)$ is obtained from each engagement directly, (see Section 5.5.10). Then

$$S_i(\varepsilon) = (1/N) \sum_{n=1}^{N} S_i(\varepsilon, n) \quad , \qquad (6.3\text{-}15)$$

and

$$S_i = \frac{\sum_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_D(\varepsilon) S_i(\varepsilon) + .5 \sum_{\varepsilon_1, \varepsilon_2} P_D(\varepsilon) S_i(\varepsilon)}{P_D(N_\varepsilon - 1)} \quad , \quad i = F, B \qquad (6.3\text{-}16)$$

since the occurrence of engagements at an ε is given by $P_D(\varepsilon)$.

In the above manner various measures of the performance of each aircraft in an encounter are obtained from the results of each engagement.

6.4 A Measure of Effectiveness

In an attempt to develop tactics employed by the combatants in an air duel, one must evaluate the performance of the combatants in light of their objectives. To evaluate this performance various measures of effectivencas are employed. Yet quite often these measures do not reflect the objectives of the combatants. For example, to maintain that the probability of killing the target is of utmost importance is to neglect the survival of the aircraft for use against future targets. On the other hand, to maximize the survival probability is to neglect the purpose of engaging the enemy. In short, what is needed here is a concept that encompasses both probabilities of kill and survival; a trade-off between the two. Such a concept is $E_{KT}^{(n)}$, the expected number of targets an aircraft will kill over a useful life of at most n sorties. The expectation involves the concept of killing the enemy while placing importance on survival so as to kill other enemies in the future. The present section derives this expectation.

Consider an aircraft which will fly n sorties if it survives the enemy defenses. (For various reasons such as maintenance, obsolescence, replacement, end of war, accidents, etc., an aircraft may not fly more than n sorties even in the absence of enemy defenses.) Let the probability that the aircraft survives the duel on the first sortie be P_S. Then the probability that the aircraft lives to fly the second sortie is P_S. By "flying a sortie" is meant that the aircraft at least starts the sortie and gets into combat. The probability that the aircraft flies the third sortie is P_S times the probability that it survives the second sortie, or P_S^2. Hence, by induction the probability that the aircraft flies the k^{th} sortie is P_S^{k-1}. Then $E_S^{(n)}$, the expected number of sorties flown in n attempts is given by

$$E_S^{(n)} = 1 + P_S + P_S^2 + \ldots + P_S^{n-1} , \qquad (6.4\text{-}1)$$

or

$$E_S^{(n)} = \begin{cases} \dfrac{1 - P_S^n}{1 - P_S} , & \text{if } P_S < 1 , \\ n , & \text{if } P_S = 1 . \end{cases} \qquad (6.4\text{-}2)$$

On each sortie let the probability the aircraft kills its enemy in the air duel be P_K. The expected number of enemies killed is

$$E_{KT}^{(n)} = P_K \, E_S^{(n)} \qquad . \qquad (6.4\text{-}3)$$

Utilizing the previously defined notation, let EKB be the expected number of bombers killed in n* attempted sorties by the fighter. Then

$$EKB = \begin{cases} PKB \dfrac{1 - P_{SF}^{n*}}{1 - P_{SF}} , & \text{if } P_{SF} < 1 , \\ n* \, PKB , & \text{otherwise} . \end{cases} \qquad (6.4\text{-}4)$$

Let EKF be the corresponding expectation for the fighter,

$$EKF = \begin{cases} PKF \dfrac{1 - P_{SB}^{n*}}{1 - P_{SB}} , & \text{if } P_{SB} < 1 , \\ n^* \, PKF , & \text{otherwise} . \end{cases} \qquad (6.4\text{-}5)$$

Under certain conditions emphasis may be placed on the probability of kill of the enemy as a relevant measure of effectiveness. Indeed, if killing the enemy is of utmost importance and the future matters not, then $n^* = 1$ and $E_{KT}^{(1)} = P_K$.

6.5 Computational Considerations

In this section the concern is with the method in which the various probabilities derived above are computed.

The individual engagement probabilities, $P_j^m(\varepsilon, n)$ are computed in the P_j^m Routine (see Volume III, page 43). In this routine the quantity $p(\ell)$ is computed as the product of $\bar{x}$, $\bar{y}$ and $P_k'(MI(\ell), i)$ where $\bar{x}$, $\bar{y}$ correspond to $q(\ell)$, $y(\ell)$, respectively as defined in Section 6.2. A further consideration is to identify the end of the list of $T(\ell)$ values. The precise number of firings, z , for a particular engagement is in fact not calculated in the program. However, prior to the engagement all values of $T(\ell)$ are set to zero. Hence, since a firing cannot occur at $t = 0$, a zero value for some $T(\ell)$ indicates the end of the list has been reached. The remaining portions of this routine are either self-explanatory or follow the derivations above.

The CALCULATE GRID DATA Routine (see Volume III, page 44) is the routine from which the P_j^m routine is called. The other purposes of the routine are

1) to distinguish between aware and unaware cases, and
2) to set the single shot kill probabilities, $P_k'(MIS, i)$, either to their input value or to zero according to the value of m.

The purpose of the CALCULATE ε DATA Routine (see Volume III, page 44) is to compute the probabilities $P_j^m(\varepsilon)$ independent of each grid-point. It, therefore, calls the CALCULATE GRID DATA Routine while iterating on the grid-point values, and performs the indicated summation. Near the end of this routine the sum is divided by the appropriate number of cases found. It is well to note that the value of 2.0 is assigned to probabilities that are undefined because no cases occurred. For example, if the bomber was unaware for all the grid-points of some value of ε, the $P_j^m(\varepsilon)$ will be set to 2 for $m = 1, 2$ and $j = 1, 2$. This is done so that such conditions may be readily identified.

Finally, the CALCULATE UNCONDITIONAL DATA Routine (see Volume III, page 45) is executed. Here the encounter probabilities and expectations are computed exactly as derived above.

The total DATA PROCESSING Model is executed by the EXECUTIVE Routine - DPM, (see Volume III, page 42). The purpose of this routine is to call the two input routines which retrieve the information from the engagements and then to iterate on the various values of ε while calling the other routines as needed.

SECTION 7

DEFINITIONS

7.1 Introduction

Section 7 is intended primarily for reference. Section 7.2 defines all symbols used in the flow charts and the text, although the symbols are also defined where introduced in the text. The units associated with a variable are included, both for clarification and for use in setting input values.

An "I" after a definition means that this variable is an input to the model. A "C" meana the variable ia internally computed. This can be useful when reading the flow charts, as some variables which abstractly seem to be "inputs" are actually computed from other values. Variables used only in the text for model discussion heve a 1) after their definition. This list is repeated in Volume III, Section 7.

Also included in this list is the FORTRAN symbol associated with each variable where appropriate. These symbols do not include the subscripts of subscripted FORTRAN symbols nor do they include the arguments of FORTRAN functions.

Section 7.3 discusses some input restrictions.

7.2 Definition List

Symbol	Definition	Fortran
A	An index with values of 0 and 1 indicating respectively that the bomber was unaware or aware during an engagement. Also used in text for the area within which the bomber flies during the search. (C)	IA
a	The area swept out by the fighter's detection pattern during its search.[1] (ft^2)	
a_i	The acceleration of aircraft i. (ft/sec^2) (C)	A
ACTIVE	A logical variable that takes values of YES and NO (or, equivalently, TRUE and FALSE) indicating whether or not an aircraft has active information from optical or detection radar. (C)	ACT
$a_{DEC}(i)$	The input deceleration of aircraft i. $a_{DEC}(i)$ must be input as a negative number. (ft/sec^2) (I)	ADEC
$A_M(i)$	The input parameter of the decreasing lag course function of aircraft i. This is the angle that aircraft i will try to leg by when its enemy is flying pure pursuit and $\lambda_i = 0$. (deg) (I)	AM
B	An index identifying the aircraft designated as "bomber." B always equals 2. (I)	IBMR
B_i	An index with values 0, 1 indicating the pilot's sickness state. (C)	BINDEX
C	A flow chart symbol used to indicate the general maneuver of Circle.	
CL	A flow chart symbol denoting a general Circle Lost maneuver; i.e., lost information.	
D	The distance traveled by the bomber during the fighter's search. (ft) (I)	D
DIV	A temporary computation used in the Data Processing Model. (C)	DIV

1) Used in text only.

Symbol	Definition	Fortran
D(k, i)	An index indicating the general maneuver to be performed by aircraft i when in information atate k ; 0 = evasive maneuver, 1 = aggressive maneuver. (1)	D
e	The base of the natural logarithm = 2.71828	EXP
EKB, EKT	The expected number of bombers killed hy a fighter over ita useful life. (C)	EKB
EKF	The expected number of fighters killed by a bomber over ita useful life. (C)	EKF
ESB	A temporary calculation of the expected number of aorties completed by the fighter. (C)	ESB
E_C	A flow chart symbol used to indicate the general maneuver of Evade by Circle.	
E_L	A flow chart symbol used to indicate the general maneuver of Evade Linearly.	
E_S	Specific energy.[1)] (ft)	
$E_S^{(n)}$	The expected number of sorties completed in at moat n attempts.[1)]	
ENV SW(MIS, i)	A switch which when ON, or TRUE, indicates that aircraft i has fired a weapon of type MIS; otherwiae the weapon type has not been fired and the variable haa a value of OFF, or FALSE. (C)	ENV SW
F	The aegment Y* times the ratio of the bomber's velocity to the relative velocity; F is used in the computation of $P_D(\epsilon)$. (ft) (C)	FSMALL
F	An index identifying the aircraft designated "fighter." F always equals 1. (C)	1FTR
f_1 , f_2	Oxygen flow leaving and returning to pilot's brain. Functional notation in text is made explicit in flow chart.[1)] (sec^{-1})	
g_1 , g_2	An arbitrary number of g's pulled.[1)]	
G_1 , G_2 , G_3	The three levels of target's total g'a for which the weapon envelopes are input. (I)	GT
$G_B(i)$	An index with values of 0, 1 indicating respectively that the degree of oxygen debt of the pilot of aircraft i will not or will affect the maneuverability of his aircraft by limiting the g's of his aircraft. (I)	GB

Symbol	Definition	Fortran
G_i	The number of total g's being sustained by aircraft i . (C)	G
$G_i(V)$	The structural or aerodynamic limit of total g's for aircraft i as a function of its velocity. (I)	GBIG
$g_i(V)$	The total g function of velocity for aircraft i at which the specific power function is zero. Note if $g_i(V) > G_i(V)$ then for that V $g_i(V)$ is unattainable and $G_i(V)$ should be inputted. (I)	GMAXT
$g_{MIS}(i)$	The level of total g's for aircraft i above which weapon type MIS cannot be fired from aircraft i . (I)	GMIS
$G_p(i)$	The maximum number of total g's that the pilot of aircraft i is able to sustain. (I)	GP
h	The altitude of the simulated engagements; used as an identifier only. (ft) (I)	H
i, j	Indices that take on values of F and B and do not have the same value. These symbols always indicate an aircraft and nothing else. (C)	I, J
$IA(\ell)$	The aircraft that fired the ℓ^{th} weapon in an engagement. (C)	IAFIRI
ICAN(i)	An index with values 1, 0 indicating respectively that aircraft i's firing of any weapon is or is not being delayed in an engagement so as to get to a better position at the time of firing. (C)	ICAN
ID	Identifying titles of the combatants for printing purposes. (I)	ID
ISHIFT	An index with values 1, 2 and 3 indicating that the first, second or third value of $t_D(\cdot)$ is assigned to Δt . (C)	ISHIFT
ITEMP(i)	An index with values 1, 0 indicating respectively that aircraft i does or does not have IFF. (C)	ITEMP
k	An index with values 1, 2, ..., 11 indicating the information state of an aircraft. (Also used throughout as an arbitrary index with integer values; when used as such it is defined in context.) (C)	K

Symbol	Definition	Fortran
k_i	The k state of aircraft i . (C)	KPRT
K_F , K_B , K	Symbols used to describe the slope of the DEL Pursuit Course function.[1]	
k(MIS, i)	A counter of the number of weapons of type MIS fired from aircraft i . (C)	KOUNTR
L	One greater than the total number of weapons that may be fired by both aircraft; $L = 1 + \Sigma_i \ \Sigma_{MIS} N(MIS, i)$	LCAP
L	A flow chart symbol used to indicate a Linear course.	
ℓ	An index giving the order in which weapons were fired in an engagement. $\ell \leq L - 1$. (C)	LLITL
m	An index with values 1, 2 indicating whether firing by one or both aircraft is permitted. Used in P_j^m , PK_j^m , etc. (C)	M1
MI(ℓ)	The MIS identification number of the ℓ^{th} weapon fired in an engagement. (C)	MISTPI
MIS	The identification number assigned to a weapon type, MIS takes values of 1, 2, ..., $n_m(i)$. (C)	MIS
m_i	An index identifying the position, velocity and information (k) state of aircraft i . (C)	IMSTAT
N	The number of grid-points or points of initialization for each ϵ . (I)	N
n	An index with values 1, 2, ..., N indicating the grid-point number under consideration. Also used in text for other purposes but always so identified. (C)	IGRIDP
NUIND	An index with values of 0, 1, 2 indicating the mode of operation of the tracking radar of each aircraft. (I)	NUIND

Symbol	Definition	Fortran
$n_m(i)$	The number of distinct weapon types carried by aircraft i. (I)	NM
$N_u(\epsilon)$	The number of grid-points in which the bomber was unaware for a given ϵ. (C)	NU
N_ϵ	The total number of ϵ values used in an encounter. (C)	NEPS
n'	An index used to compute the variables $\bar{x}$ and $\bar{y}$. (C)	NT
n^*	The maximum number of sorties by each aircraft, for the calculation of EKB and EKF. (I)	YN
N(MIS, i)	The total number of weapons of type MIS on aircraft i. (C)	NUMIS
O_i	A variable indicating the amount of "oxygen debt" of the pilot of aircraft i incurred by pulling g's over a period of time. (C)	OXDEBT
P	A flow chart symbol used to indicate the general maneuver of Pursuit.	
P	Represents "pursuer" in text (often used as subscript).[1]	
PASSIVE	A logical variable that takes values of YES and NO or TRUE and FALSE indicating whether or not an aircraft has passive information. (C)	PASS
PKB	The probability for an encounter that the bomber is detected and killed. (C)	PKB
PKBGD	The probability that the bomber is killed given detection for the encounter. (C)	PKBGD
PKE	The probability that the bomber is detected and killed at or before the time it becomes aware for an encounter. (C)	PKE
PKF	The probability for an encounter that the fighter detects the bomber and the fighter is killed. (C)	PKF
PKFGD	The probability that the fighter is killed, given detection for the encounter. (C)	PKFGD

Symbol	Definition	Fortran
PKG	The probability that the bomber is killed after it becomes aware, given that it survived to the time of its awareness for an encounter. (C)	PKG
PKL	The probability that the bomber is killed after it becomes aware and it survived to the time of its awareness for an encounter. (C)	PKL
P_D	The probability of detection of the bomber. (C)	PDD
P_S	Specific power.[1)] (ft/sec).[1)] Also probability of survival of an aircraft.[1)]	
P_{SB}	The probability for an encounter that the bomber survives. (C)	PSB
P_{SF}	The probability for an encounter that the fighter survives and detects the bomber and the bomber is aware of the fighter. (C)	PS
P_u	The probability that the bomber is unaware for an encounter. (C)	PUU
$P(\ell)$	The probability that the target is dead just after the ℓ^{th} weapon hits its target in some engagement. (C)	P
$p(\ell)$	The probability an aircraft is killed by the ℓ^{th} weapon only.[1)]	
PS(i)	An index indicating the capability of the passive receiver of aircraft i ; 0 implies no capability, 1 implies the ability to detect the presence of another aircraft but not the position, and 2 implies the capability of 1 with the ability to distinguish the hemisphere of the source. (I)	IPS
$P_1(j)$	An index that when set to zero will require aircraft j to turn its tracking radar on for one time pulse only when launching a weapon. (I)	P1
$P_2(i)$	An index with values 1 and 0 indicating respectively that aircraft i has or has not activated its tracking radar. (C)	P2
PV(x)	A function that gives the principal value of its angular argument. (rad) (C)	PV
P(x, y)	Probability distribution of bomber's (x, y) coordinates during search.[1)]	

Symbol	Definition	Fortran
$P_{U/D}$	The probability that the bomber is unaware, given that it is detected for an encounter. (C)	PUPD
$P_D(\varepsilon)$	The probability of detection of the bomber by the fighter for some ε. (C)	PD
$P_U(\varepsilon)$	The probability that the bomber is unaware of the fighter for some value of ε. (C)	PU
$P_i(V, \dot{\beta})$	The specific power function of aircraft i at velocity V and turning rate $\dot{\beta}$. (ft/sec) (I)	PEEU
PK_B°	The probability that the bomber is killed, given that the bomber is detected and unaware for an encounter. (C)	PK2B
PK_j^1	The encounter probability that aircraft j is killed, given that the bomber is detected and aware and that aircraft j does not fire. (C)	PKK
PK_j^2	The encounter probability that aircraft j is killed, given that the bomber is detected and aware. (C)	PKK
$P_B^\circ(\varepsilon)$	The probability for a given ε that the bomber is killed, given that it is detected and unaware.	PZ
$P_j^1(\varepsilon)$	The probability for a given ε that aircraft j is killed, given that the bomber is detected, aware and that aircraft j does not fire. (C)	PK
$P_j^2(\varepsilon)$	The probability for a given ε that aircraft j is killed, given that the bomber is detected and aware. (C)	PK
$P_C(\varepsilon)$	The probability that the bomber is killed at or before the time it becomes aware for some ε. (C)	PCCEPS
$P_B^\circ(\varepsilon, n)$	The probability for a given ε and grid-point n that the bomber is killed, given that it is detected and unaware. (C)	PPZ
$P_j^1(\varepsilon, n)$	The probability for a given ε and grid-point n that aircraft j is killed, given that the bomber is detected and aware and that j does not fire. (C)	PP
$P_j^2(\varepsilon, n)$	The probability for a given ε and grid-point n that aircraft j is killed, given the bomber is detected and aware. (C)	PP

Symbol	Definition	Fortran
$P_C(\varepsilon, n)$	The probability that in an engagement the bomber is killed at or before the time it becomes aware. (C)	PCCEPN
$P'_k(MIS, i)$	The probability of kill of weapon type MIS on aircraft i. (I)	PKP
$p_k(MIS, i)$	The probability of kill of weapon type MIS on aircraft i; set to the input value, $P'_k(MIS, i)$, or to zero. (C)	PK
Q_B	The quadrant of the point B on the fighter's detection pattern. (C)	QB
Q_C	The quadrant of the point C on the fighter's detection pattern. (C)	QC
$q(\ell)$	The probability both aircraft are alive at time $T(\ell)$.[1]	
$Q(X)$	A function that gives the quadrant of the angle X. (C)	Q
R	The range between the two aircraft. (ft) (C)	R
r	The range of the detection capability of the fighter; r is set to $R_{DET}(F)$ if this is not zero and to $R_{OPT}(F)$ otherwise. Also used in text for the Y* projection against a stationary target. (ft) (C)	RSMALL
R_1	A variable used to indicate whether the tracking radar is turned on. (ft) (C)	R1
R_1, R_2	Distances used in describing steady state.[1] (ft)	
r_{in}	An override initial range that will act so as to shrink the detection range to r_{in}. (ft) (I)	RANGE
$R'(1, i)$, $R'(2, i)$	The first and second values respectively that will be assigned to $R^*(i)$, i.e., before and after the opponent becomes aware. (ft) (I)	RPRIME
RFLOOR(MIS, i)	A superimposed minimum boundary of weapon type MIS such that the weapon type may not be fired from aircraft i whenever the range is less than RFLOOR(MIS, i). (ft) (I)	RFLOOR

Symbol	Definition	Fortran
RNOW(i)	The range at which aircraft i may commence to fire at an unaware enemy; against an unaware target, firings by aircraft i are postponed until within a range of $R \leq$ RNOW(i) . (ft) (C)	RNOW
RTEST(1) , RTEST(2)	The ranges at which Δt will change values from $t_D(1)$ to $t_D(2)$ and from $t_D(2)$ to $t_D(3)$ respectively. (ft) (1)	RTEST
$R_{PAS}(i)$	The range of the passive detection capability of aircraft i , (ft) (1)	RPAS
$R_{TRK}(i)$	The range of the tracking radar of aircraft i , (ft) (1)	RTRK
$R_{OPT}(i)$	The range of the optical capability of aircraft i , (ft) (1)	ROPT
$R_{DET}(i)$	The range of the detection radar of aircraft i , (ft) (1)	RDET
$R_{IFF}(i)$	The range of the IFF capability of aircraft i , (ft) (1)	RIFF
R_{min}	A minimum range which will terminate an engagement; $R < R_{min}$ causes termination of an engagement. (ft) (1)	RMIN
R'_{MIS}	The minimum range of some weapon type. (ft) (C)	RMISP
R_{MIS}	The maximum range of the weapon envelope of some weapon type. (ft) (C)	RMIS
$R_1(V_a, o_b,$ MIS, i)	The outer weapon envelope of weapon type MIS on aircraft i associated with a velocity of V_a , an angle-off of o_b for a value of target g's of G_1 . (ft) (I)	RF1T
$R_2(V_a, o_b,$ MIS, i)	The same as $R_1(V_a, o_b,$ MIS, i) but for a target g level of G_2 . (ft)(I)	RF2T
$R_3(V_a, o_b,$ MIS, i)	The same as $R_1(V_a, o_b,$ MIS, i) but for a target g level of G_3 . (ft) (I)	RF3T
$R'_1(V_a, o_b,$ MIS, i)	The inner envelope limit of weapon type MIS on aircraft i for an average velocity of V_a and angle-off of o_b for G(1) total target g's. (ft)(I)	RF1PT
$R'_2(V_a, o_b,$ MIS, i)	The same as $R'_1(V_a, o_b,$ MIS, i) but for a target g level of G_2 . (ft) (I)	RF2PT
$R'_3(V_a, o_b,$ MIS, i)	The same as $R'_1(V_a, o_b,$ MIS, i) but for a target g level of G_3 . (ft) (I)	RF3PT

Symbol	Definition	Fortran
$\dot{R}$	The rate of change of the range; (ft/sec) (C) $$\dot{R} = dR/dt \quad .$$	RDOT
$R^*(i)$	The range which aircraft i will attempt to attain when in the rear of its enemy. (ft) (C)	RSTAR
$R_g(i)$	The range of the tail gun of aircraft i . (ft) (I)	RANGUN
$R(\phi_j)$	The maximum range at which aircraft j can fire any weapon when approaching an unaware target from the rear, based on the speeds of the aircraft and tracking angle of the pursuer. It assumes the target flies linearly. This is also used as the name of the routine that calculates $R(\phi_j)$. (ft) (C)	RPHIJ
S_i	The expected fraction of t_{MAX} that the pilot of aircraft i will spend in a "sick" condition, $0_i > 1$, for an encounter. Also used to represent the total amount of time that the pilot of aircraft i spends in a sick condition. (In latter case: sec) (C)	TIMSIC
$S_i(\varepsilon)$	The expected fraction of t_{MAX} that the pilot of aircraft i will spend in a "sick" condition for some value of ε . (C)	SICEPS
$S_i(\varepsilon, n)$	The fraction of t_{MAX} that the pilot of aircraft i spends in a "sick" condition during an engagement defined by ε and n . (C)	SICTIM
ST(i)	An index indicating the general maneuver of aircraft i ; (C) 0 ⟶ circle 1 ⟶ linear flight 2 ⟶ pursuit course 3 ⟶ circle, lost information 4 ⟶ evade 5 ⟶ evade, lost information	IST
S	An estimate of the amount of change in range when decelerating at a constant rate from some velocity to V_0 at a rate of $a_{DEC}(i)$. (ft) (C)	S
sgn(x)	The signature function of the argument x ; (C) $$\text{sgn}(x) = \begin{cases} 1, & \text{if } x \geq 0, \\ -1, & \text{Otherwise.} \end{cases}$$	SGN

Symbol	Definition	Fortran
$S_f(i)$	An index with values of 0, 1 indicating respectively that the degree of oxygen debt of the pilot of aircraft i will not or will effect the firing of weapons by retarding such firings. (I)	SF
S(MIS, i)	An index with values of 1 and 0 respectively indicating that aircraft i has or has not fired weapon type MIS. (C)	S
T	Represents "target" in text (often used as subscript).[1]	
t	The amount of time since initialization of an engagement. (sec) (C)	T
t_f	The time of flight of all weapons. (sec) (I)	TF
t'	An arbitrary time prior to T(ℓ).[1] Also used in text as a time prior to detection. (sec)	
t_{AWARE}	The time at which the bomber became aware of the presence of the fighter. (sec) (C)	TAWARE
t_{LAST}	The duration of time of an engagement. (sec) (C)	TLAST
t_{max}	The maximum amount of combat time allowed for a single engagement. (sec) (I)	TMAX
t_{min}	The amount of time required to elapse after which a loss of information by both combatants will terminate an engagement. (sec) (I)	TMIN
t_{PRT}	The amount of time until the next printout of each aircraft's relevant parameters. (sec) (C)	TPRT
$t_D(1)$, $t_D(2)$, $t_D(3)$	The first, second and third values assigned At at various transitions during an engagement. (sec) (I)	TDELTS
$t_C(i)$	The maximum amount of combat time allowed for aircraft i. (sec) (I)	TC
$t_{MIS}(i)$	The time at which aircraft i fired the first weapon of type MIS. (sec) (C)	TMIS
t(MIS, i)	The time of the last firing of a weapon of type MIS from aircraft i. (sec) (C)	TLASTF
t^*	The amount of time between printouts of the combatants' relevant variables. (sec) (I)	TSTAR

Symbol	Definition	Fortran
$T(\ell)$	The time at which the ℓ^{th} firing took place during an engagement. (sec) (C)	TFIREI
V_a	Aircraft speed used in the input tables for launch envelopes and for energy-maneuverability. (ft/sec) (I)	VATAB VATAG
V_o	The speed for aircraft i which would make the range rate, R, equal zero, subject to $V_o \geq V_C(i)$. (ft/sec) (C)	VZERO
V'	An arbitrary speed used in the distance function S.[1] (ft/sec)	
V*	The speed of one aircraft relative to the other at the time of initialization. (ft/sec) (C)	VSTAR
V_i	The speed of aircraft i. (ft/sec) (C)	V
$V_o(i)$	The initial speed of aircraft i. (ft/sec) (I)	VZ
V*(i)	The speed at which the sustainable turning rate of aircraft i is an absolute maximum. (ft/sec) (C)	VSTR
$V_C(i)$	The minimum sustainable speed of aircraft i. (ft/sec) (C)	VC
$V_{max}(i)$	The maximum sustainable speed of aircraft i. (ft/sec) (I)	VMAX
W_i	The weight of aircraft i. (lbs) (I)	W
X	One axis of the moving coordinate system used in the initiation phase.[1] (ft)	
XPHI	A multiplier with values 0 and 1 to change the computed angle ϕ^* to 0 or to leave it as is. (I)	XPHI
x_i	The x position of aircraft i in the (x, y) inertial coordinate system. (ft) (C)	X
x	A symbol used as the argument of various functions. Also used for one axis of the inertial coordinate system.[1] (ft in latter case)	
x	Used for temporary computations. (C)	CAPX

Symbol	Definition	Fortran
x_1	A temporary calculation of the maximum turning rate of an aircraft when pulling the minimum of $g_i(V_i)$ and $G_i(V_i)$ total g's. (rad/sec)[2]	
x_2	A temporary calculation of the maximum turning rate of an aircraft when pulling $G_i(V_i)$ total g's. (rad/sec)[2]	
$\bar{x}$	A temporary calculation of the probability that both aircraft are alive at the time of firing of the ℓ^{th} weapon. (C)	X
x_i', y_i'	Temporary computations of the future position of aircraft i. (ft)[2]	
Y	One axis of the moving coordinate system used in the initiation phase.[1] (ft)	
y	One axis of the inertial coordinate system.[1] (ft)	
Y_g	The relative Y coordinate of the bomber at the beginning of the engagement. (ft) (C)	YG
Y_{MAX}	The upper limit of the segment Y* in the (X, Y) coordinate system (used in initiating engagements). (ft) (C)	YMAX
Y_{MIN}	The lower limit of the segment Y* in the (X, Y) coordinate system (used in initiating engagements). (ft) (C)	YMIN
Y*	The normal projection of the fighter's detection pattern onto the Y axis in the (X, Y) coordinate system at the beginning of one engagement. (ft) (C)	YSTAR
y*	The steady state speed. (ft/sec) (C)	YNEW
$\bar{y}$, $y(\ell)$	A temporary calculation of the probability that aircraft j survived any weapons that hit between the firing and arrival of the ℓ^{th} weapon. (C)	Y
Y_B	A parameter used in determining the detection contour for some ε. (ft) (C)	B
Y_C	a parameter used in determining the detection contour for some ε. (ft) (C)	C

2) Used in flow chart only.

Symbol	Definition	Fortran
y_i	The y position of aircraft i in the (x, y) inertial coordinate system. (ft) (C)	Y
Z_K , Z_K' , Z_L , Z_L'	Temporary calculations used to compute the weapon launch envelopes. $(ft)^2$ 2)	
α_1 , α_2 , α_3 , α_4	Used to describe DEL course.1) (rad)	
$\dot{\alpha}_i$	The rate of change of α_i ; (rad/sec). (C) $$\dot{\alpha}_i = d\alpha_i/dt \quad .$$	ALPDOT
$\alpha_g(i)$	The half-angle defining the tail gun capability of aircraft i measured from the tail of the aircraft. (deg) (I)	ALPGUN
$\alpha_{MAX}(i)$	The internally computed parameter of the DEL function of aircraft i . (rad) (C)	ALPMAX
$\alpha_{MIS}(i)$	The half-angle of the cone in which aircraft i must be tracking its enemy in order to fire weapon type MIS. (deg) (I)	ALPMIS
α_i	The tracking angle of aircraft i , measured from the inner line of sight to the velocity vector of the aircraft. (rad) (C)	ALPHA
$\alpha_{DET}(i)$	The half-angle of the detection radar cone of aircraft i . (deg) (I)	ALPDET
$\alpha_{IFF}(i)$	The half-angle of the IFF capability of aircraft i . (deg) (I)	ALPIFF
$\alpha_{OPT}(i)$	The half-angle of the optical capability of aircraft i . (deg) (I)	ALPOPT
$\alpha_{PAS}(i)$	The half-angle of the passive detection capability of aircraft i measured off the tail of the aircraft. (deg) (I)	ALPPAS
$\alpha_{TRK}(i)$	The half-angle of the tracking radar cone of aircraft i . (deg) (I)	ALPTRK
$\dot{\beta}_i$	The turning rate of aircraft i . (rad/sec) (C)	BETDOT
β_i	The angle of aircraft i's heading measured from the x axis. (rad) (C)	BETA
γ	An angle used in defining the initial positions of the aircraft. (rad) (C)	GAMMA

2) Used in flow chart only.

Symbol	Definition	Fortran
δ	The density of fighters assigned to an area for search purposes; used in computing the probability of detection, $P_D(\varepsilon)$. (ft^{-2}) (I)	DELTA
$\Delta\alpha$	The change in α in one time pulse.[1] (rad)	
Δt	The incremental time slice of simulation -- a time for which rate of change, $\dot{R}$, $\dot{\theta}$, etc., are assumed to be constant in the integration of the equations of motion. (sec) (C)	DELTAT
ΔY_g	The change in the value of Y_g for different grid-points. (ft) (C)	DELTYG
$\Delta\varepsilon$	The change between ε values for various sets of grid-points. If input to a value greater than 180° the program will execute only one value of ε . (deg) (I)	DELEPS
ε	The angle between the velocity vectors of the combatants measured from the fighter to the bomber at the time of initialization. Also used to indicate the index of a particular value of ε in summations. (In special cases this may be an input.) (deg) (C)	EPSLON
ε_1 , ε_2	The first and last values respectively of ε . (deg) (C)	EPSLON
η	The desired tracking angle of an aircraft as computed by the DEL course function. (rad) (C)	ETA
λ_i	The constant deviate angle flown by aircraft i when inside the ϕ^* cone of its enemy. (deg) (I)	LAMBDA
θ_B	The angle between the X-axis and the ray from (0, 0) to the point B of the fighter's detection pattern. (rad) (C)	THETB
θ_C	The angle between the X-axis and the ray from (0, 0) and the point C on the fighter's detection pattern. (rad) (C)	THETC
$\dot{\theta}$	The turning rate of the line of sight. (rad/sec) (C)	THEDOT
μ	The angle between the vector V^* and the heading of the fighter at initialization. (rad) (C)	EMU
π	3.14159 ... (C)	PI

Symbol	Definition	Fortran
ρ	The half-angle of the fighter's detection capability; ρ is set to $\alpha_{DET}(F)$ if this is not zero and to $\alpha_{OPT}(F)$ otherwise. (rad) (C)	RHO
σ_b	An angle indicating the angle-off times the sign of the direction of turn of the target. (rad) (I)	SIGB
$\sigma(MIS, i)$	A counter of the amount of time that aircraft i has spent in the envelope of weapon type MIS since the last firing of weapon type MIS. (sec) (C)	SIGMIS
τ_i	The approximate amount of time that aircraft i will need to fire a weapon after its tracking radar is activated; used to compute distance from $R(\phi_i)$ at which to activate its tracking radar. (sec) (I)	TAU
$\tau(MIS, i)$	A counter of the total amount of time that aircraft i has spent in the envelope of weapon type MIS. (sec) (C)	TAUMIS
ϕ_i	The angle off the target of aircraft i ; measured from the outer line of sight to the heading of aircraft j , $j \neq i$. (rad) (C)	PHI
$\dot{\phi}_i$	The rate of change of ϕ_i ; (rad/sec) (C) $\dot{\phi}_i = d\phi_i/dt$.	PHIDOT
ϕ^*	The half-angle of the cone in the rear of a target aircraft within which pure pursuit is the navigation doctrine, if $\lambda_i = 0$. (rad) (C)	PHISTR

7.3 Input Considerations

While ATAC-2 was developed with a general user in mind, completely arbitrary inputs are, of course, impossible to handle. Limitations exist and considerntions must be given to the values of inputs. These considerations are necessary, in some instances, due to the nature of air-to-air combat and in other instances due to the specifics of the program as it exists. Below are listed some of these considerations and limitations under various headings.

7.3.1 Parameter Guidance

(1) In the program the value of the internal variable $R^*(F)$ is initially set to the input $R'(1, F)$. This is the range which the fighter tries to attain off the tail of an unaware bomber, before firing any weapons. If the fighter can maintain surprise $R'(1, F)$ will be the range at which the bomber becomes aware by being fired upon. Two considerations, therefore, should be observed when selecting a value for $R'(1, F)$.

Firstly, this range should be such as to allow firings of the more lethal weapons of the fighter. Secondly, the range should be such as to allow the fighter to stay behind the bomber when the latter becomes aware and begins to maneuver. This last consideration is very difficult to quantify. In general, the ability of the fighter to stay behind a maneuvering bomber is a function of both velocities, both turning rates and both deceleration rates. However, the closer the fighter is to its steady state range and its associated velocity (see Appendix F), if it exists, the easier it will be to stay behind the bomber.

(2) The range $R^*(i)$, input as $R'(2, i)$, represents the range at which aircraft i would like to be off the tail of its enemy so as to be able to fire its shortest range weapons, usually guns, while the enemy is maneuvering. The selection of this parameter is influenced by the capability of this shortest range weapon. However, the maneuverability of both aircraft must be accounted for. In Appendix F it is shown that selection of $R^*(i)$ (or a steady state range) has implications on other parameters as well, namely the velocity and angle-off associated with $R^*(i)$. Thus, if possible these considerations (the values of $R^*(i)$ that allow weapon firings and the values of $R^*(i)$ that allow steady state conditions to obtain) should be combined to arrive at a realizable $R^*(i)$ whenever possible.

It should be noted that $R^*(i)$ originally takes on an input value $R'(1, i)$ which in the case of the fighter, is the range at which it may begin firing. Once this range is achieved, it then closes to this close range $R'(2, i)$.

(3) The length of the time pulse Δt influences the running time of the computer program. The running time is inversely proportional to Δt . However, as Δt increases some singularities occur. For example, the path of an aircraft may cross a launch envelope from one pulse to the next, without being in the launch envelope at the beginning of a pulse. A possible firing will be missed. Also, since the method of numerical integration in the model assumes that the time derivatives of the relative parameters are constant for a period of time Δt , the length of Δt should be kept fairly small. The error due to this assumption is inversely proportional to R^2 . The assumption is therefore worse for

smaller values of range; hence the capability in the program to decrease the length of Δt as the range gets smaller. Typical values for Δt are:

$t_D(1) = 5.0$ sec. $\quad R_{TEST}(1) = 100{,}000$

$t_D(2) = 2.0$ sec. $\quad R_{TEST}(2) = 70{,}000$

$t_D(3) = .25$ sec.

7.3.2 Model Logic

The following are logical restrictions of the model, i.e., the violation of them will affect the logic of the model:

1. All calculations involving $a_{DEC}(i)$ assume an associated negative value; hence $a_{DEC}(i)$ must be inputted as less than or equal to zero.

2. IFF is necessary before firings can take place; $\alpha_{IFF}(i)$ and $R_{IFF}(i)$ must, therefore, be non-zero if aircraft i is to fire its weapons.

3. Each aircraft may obtain active information only from either its detection radar or optical system. An extension of tracking radar coverage over that of the detection coverage adds no capability. To avoid confusion and false interpretation the tracking pattern should be contained within the detection pattern: $\alpha_{TRK}(i) \leq \alpha_{DET}(i)$ and $R_{TRK}(i) \leq R_{DET}(i)$.

5. Whenever $g_i(V)$ is unattainable due to the value of $G_i(V)$ being prohibitive, any value greater than or equal to $G_i(V)$ may be used for $g_i(V)$ at that V. The program will select the minimum of the two when appropriate.

6. The tail gun of aircraft i is assumed to be weapon number $n_m(i)$, the last weapon. Further $n_m(i)$ may not be zero. The capability and hence the effect of the tail gun may be negated by setting $R_g(i)$ and $\alpha_g(i)$ to zero, or setting $N(n_m(i), i)$ to zero.

7.3.4 Program Restrictions

The following are restrictions that exist in the current program. They arise from computer space allocation considerations. To change them, however, may require significant programming effort.

1. The number of weapon types must be no more than six: $n_m(i) \leq 6$.

2. The number of grid-points for one ε must be limited by twenty: $N \leq 20$.

3. The number of speed values for which the weapon launch envelopes are input must be either 2 or 3.

4. The number of angular values for which the weapon launch envelopes are input must be between two and fifteen inclusive.

5. Since the number of ε values considered will be $180°/\Delta\varepsilon$ or $360°/\Delta\varepsilon$, $\Delta\varepsilon$ must be greater than or equal to $30°$ if PS (F or B) = 1 and greater than or equal to $15°$, otherwise.

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SECTION 1

INTRODUCTION

Consider a fighter searching an area for enemy aircraft. The fighter's purpose is to detect, engage and destroy the enemy. When an enemy is detected the fighter tries to maneuver to an advantageous position from which to press the attack, launch its weapons and accomplish the mission unscathed.

The development of the Double Search Mode was prompted by an interest in the case in which the enemy is also a fighter on search for the friendly aircraft. The bulk of the model is the same as the Single Search Mode[1)] with one major conceptual change.

The Single Search Mode of ATAC-2 treats combat that arises when a fighter searches for a bomber. The bomber is docile until it becomes aware that an enemy aircraft is attacking. Thereafter the bomber becomes aggressive as well.

A characteristic of the Single Search Mode is, then, that the fighter must detect the bomber for combat to take place. But while the outcome of air-to-air combat depends on many factors, generally the aircraft which detects first gains a decided advantage. The terms of the ensuing battle are usually more readily controlled by that aircraft which detects first. Thus the Double Search Mode requires changing the Single Search Mode to include a method by which either aircraft could first detect its enemy.

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1) Harris, T. J., B. J. Jacobs, W. J. Strauss, Fighter Vs. Fighter Combat: ATAC-2 Model: Single Search, Volumes I, II, III and IV, Caywood-Schiller, Associates, September, 1967.

It is insufficient simply to run the Single Search Mode twice, once with one of the aircraft used as the fighter, and once with the other aircraft as the fighter. Cases would arise when the "bomber" should detect first, but does not react immediately. Some of the kill probabilities, therefore, would be different.

Once detection occurs, the model traces through the ensuing sequence of events by means of a deterministic, time slice simulation. No Monte Carlo processes are involved. The model treats the two aircraft alike, subject only to the constraints inputted for each aircraft. At any particular moment in time, an aircraft may be flying straight, be in a turn, or be on a form of pursuit course. It may be flying at constant speed, accelerating or decelerating. Both aircraft (usually) attempt to maneuver themselves into positions to fire their weapons. The model is dynamic in that at any given time the maneuvers performed, and the times at which weapons are fired by each aircraft, depend on the relative position of the aircraft. Further, the maneuver and weapon firings depend upon the information each aircraft has about the position and activity of the other aircraft at the time. The information made available to an aircraft depends on the sensors specified for it.

The Double Search Mode consists of two major submodels: The ENGAGEMENT Model (EM) and the DATA PROCESSING Model (DPM). The EM answers questions such as: Can either aircraft fire its weapons? If yes, when, which weapons and how often? The EM produces a time ordered sequence of events that occur during a dogfight. After detection occurs, the EM of the Double Search Mode is identical to that of the Single Search Mode. The DPM transforms the sequence into overall battle outcomes such as the probability of kill of an aircraft. The Double Search Mode engenders minor changes in the DPM, but the model is essentially the same.

SECTION 2

DETAILS

2.1 Introduction

As indicated previously the main difference between the Single and Double Search Modes is the fact that in the latter either aircraft may detect initially, thus playing the role of the fighter as defined in the Single Search Mode. This is a direct consequence of the assumption that each aircraft searches for the other in an aggressive manner. However, tactics within an engagement remain unchanged between these modes, since the tactics were designed the same for both aircraft in the duel. Only the manner in which engagements are initialized need be changed in the ENGAGEMENT Model. Changes are required also in the DATA PROCESSING Model. To account for detection by either aircraft the Single Search detection procedure is exercised twice, once for each aircraft. Then two points of possible detection are obtained; one for the first aircraft detecting the second and one for the second detecting the first. Then the detection point selected to start the engagement is the one that occurs first in real time.

One further generalization in the detection phase is the following: In the Double Search Mode an aircraft may detect either by optical sighting or by search radar. This is a feature distinct from the basic generalization of the Single Search Mode. However, the added computation requirements are not severe. What it requires is four possible detection points -- two by each aircraft, one by radar and one by visual means. The method is still simple. The model selects that detection point which occurs first in real time and starts the engagement there.

The changes in the DPM are a consequence of the manner in which an engagement starts. In this report, the event of either aircraft detecting the other together with the ensuing combat is called a duel. For convenience a duel conditional on a particular ε or grid-point is called an engagement. A duel unconditional on ε is called an encounter. In the Single Search Mode a duel starts when aircraft number 1 detects aircraft number 2. The probability a duel occurs is the probability aircraft number 1 detects aircraft number 2. In the Double Search Mode, however, an engagement starts when either aircraft detects the other. The probability a duel occurs is then the probability that either aircraft detects the other. Thus, the DPM of the Double Search Mode calculates the probability of a duel instead of the probability of detection by aircraft number 1 as in the Single Search Mode.

Before considering more detailed aspects of the Double Search Mode a word of caution regarding the subscripts F and B is called for: in the Single Search Mode both the fighter (F) and the Bomber (B) are always the same aircraft for one run of the model. The labels F and B are set to numerical constants in this mode to designate aircraft number: $F = 1$ and $B = 2$. However, this is not the case in the Double Search Mode. The detecting aircraft may be either number 1 or number 2. The labels F and B must, therefore, be redefined. Accordingly, F is defined as that aircraft which detects the other first in a certain engagement, and B is that aircraft which is detected in the engagement. For example, in one engagement F may have the value of 2, while B has the value 1, indicating that the number 2 aircraft detected the number 1 aircraft initially. It is to be noted that F and B may change values from one engagement to another in the Double Search Mode. Reference to a particular aircraft is made only by way of aircraft numbers 1 and 2.

2.2 Definitions and Redefinitions

In this section the definitions of new symbols required for the Double Search Mode are given. Also, symbols used differently here are redefined. The method of presentation is the same as the method used in Volume II, Section 7 of the Single Search Model [Ref. 1]. Previously used symbols which correspond to new symbols are shown parenthetically.

Item New (Old)	Definition	Fortran
B (B)	The name given to whichever aircraft (1 or 2) is detected initially in an engagement. (C)	IBMR
F (F)	The name given to whichever aircraft (1 or 2) initially detects in an engagement. (C)	IFTR
IE(n)	A code to identify the aircraft that detected initially for the n^{th} grid-point of ϵ. (C) IE(n) = 0 ⟶ both detect at the same time IE(n) = 1 ⟶ 1 detects 2 first IE(n) = 2 ⟶ 2 detects 1 first	IE
k	An index indicating the k^{th} detection pattern. (C) k = 1 ⟶ radar of aircraft 1 k = 2 ⟶ optics of aircraft 1 k = 3 ⟶ radar of aircraft 2 k = 4 ⟶ optics of aircraft 2	K
$N_j(\epsilon)$	The number of grid-points at ϵ in which the j^{th} aircraft detected first or simultaneously. (IE(n) = 0, j). (C)	NJ
$P_D(j, \epsilon)$	The probability that aircraft j detects initially, given an engagement at ϵ. (C)	PDJ
$P_D(j)$	The probability that aircraft j detects initially, given an encounter. (C)	PDI

Item	Definition	Fortran
New (Old)		
$P_E(\varepsilon)$ $(P_D(c))$	The probability of an engagement at ε. (C)	PE
P_E (P_j)	The probability of an encounter. (C)	PEE
$P_j(c, n)$	The probability aircraft j is killed, given an engagement at ε, n. (C)	PP
$P_j^1(c)$ $(P_j^2(c)+P_B^{\bullet}(c))$	The probability that aircraft j is killed, given an engagement at c. (C)	P
$P_j^2(c)$	The probability that aircraft j is killed, given an engagement at ε in which that aircraft j detects initially. (C)	P
PK_j^1 $(PK_j^2 + PK_B^{\bullet})$	The probability that aircraft j is killed, given an encounter. (C)	PKK
PK_j^2	The probability that aircraft j is killed, given j detects initially. (C)	PKK
r_k (r)	The range of the k^{th} detection pattern. (ft) (C)	RK
$\bar{R}_k$ (R)	The range between aircraft when detection takes place on the k^{th} pattern. (ft) (C)	RBAR
Y^* (Y^*)	The length of the segment generated by the projections of the four search patterns onto the Y-axis at a given c. (ft) (C)	Y*
$Y_{max}(k)$ (Y_{max})	The upper limit of the Y projection of the k^{th} pattern at a given c. (ft) (C)	YMAX
$Y_{min}(k)$ (Y_{min})	The lower limit of the Y projection of the k^{th} pattern at a given c. (ft) (C)	YMIN
$Y_B(k)$ (Y_B)	A control parameter used to establish the points of detection on the k^{th} pattern at a given c. (C)	YB
$Y_C(k)$ (Y_C)	A control parameter used to establish the points of detection on the k^{th} pattern at a given ε. (C)	YC
Y_{SUP}	The upper limit of the Y^* segment. (ft) (C)	YSUP
Y_{GLB}	The lower limit of the Y^* segment. (ft) (C)	YGLB

Item		Definition	Fortran
New	(Old)		
$\bar{\alpha}_k$	(α_i)	The tracking angle of the relevant aircraft when detection occurs on the k^{th} pattern. (rad) (C)	ALPBAR
ϵ'_i	(ϵ)	The angle between initial velocity vectors measured from aircraft i to j. (rad) (C)	EPSSUB
μ_i	(μ)	The angle between the initial velocity vector of aircraft i and the relative velocity vector $\vec{V}^*$. (rad) (C)	FMU

2.3 Discussion of Single Search Symbols

In this section some of the differences between variables of the Single Search Mode and the Double Search Mode are highlighted. The variables relate mainly to the generalization of the original detection process. Throughout this section Figures 2.3-1, 2.3-2 and 2.3-3 will be helpful. Figure 2.3-1 is for the case in which aircraft 1 is (tentatively) treated as the detecting aircraft, whose detecting capability is to be ascertained. V* is relative to aircraft number 1 as shown in the velocity vector diagram at the left side of the figure. The right side shows the detection pattern (only one for simplicity) of aircraft number 1 in the same position, with the (X, Y) coordinate system superimposed.

Figure 2.3-2 is for the same relative geometry except that in this case aircraft number 2 is (tentatively) treated as the detecting aircraft, and aircraft number 1 as being docile. The top vector diagram at the left of the figure is the same as in Figure 2.3-1, except that V* in Figure 2.3-2 is relative to aircraft number 2, and is, therefore, the negative of that in Figure 2.3-1. By rotating the top left diagram 180° in the plane of the figure, the bottom left diagram is obtained. The V* is now in the standard position to superimpose the (X, Y) coordinate system. The right side of the figure shows the detection pattern (only one for simplicity) of aircraft number 2 in the same position as in the bottom left diagram. The (X, Y) coordinate system is superimposed.

2.3.1 F, B and IE(n)

As was earlier stated the subscripts F and B are here "variables," set respectively to the aircraft that detected first and the aircraft which

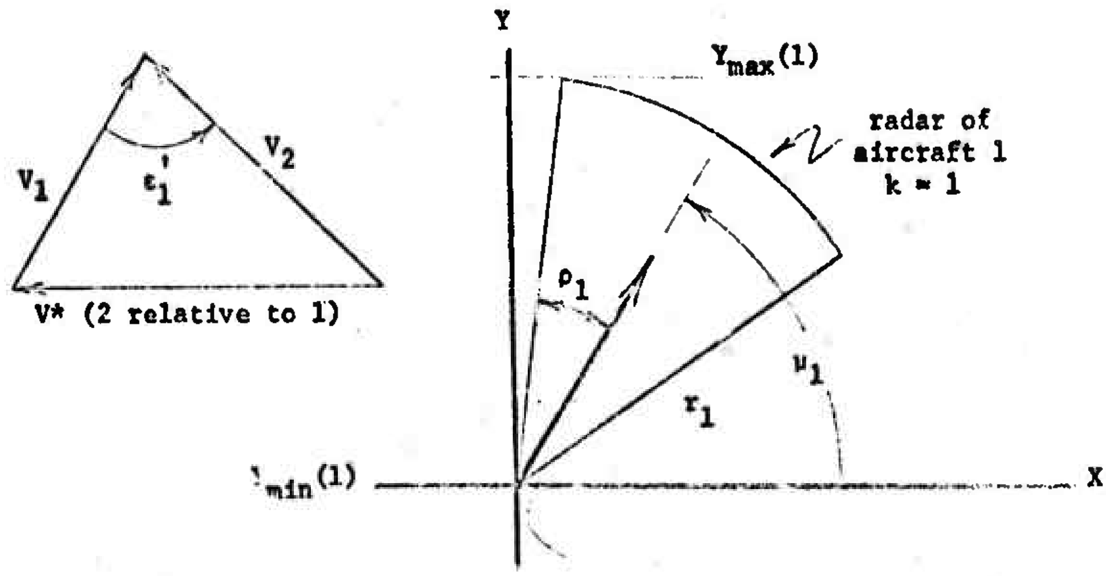


Figure 2.3-1 Detection Parameters of Double Search Mode (2 relative to 1)

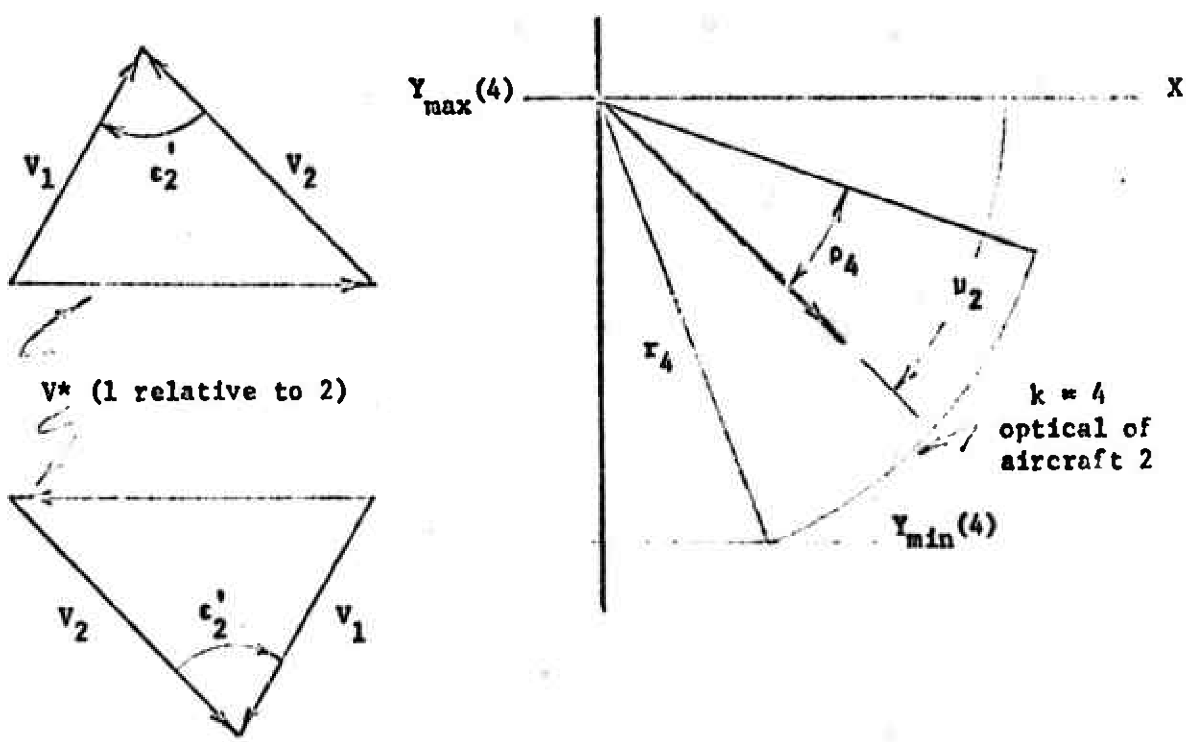


Figure 2.3-2 Detection Parameters of Double Search Mode (1 relative to 2)

was detected -- the aggressor and the surprised. For one engagement aircraft number 1 may have detected first: $F = 1$, $B = 2$. However, this may change for the next or some other engagement. To record this information the variable IE(n) is set to the identification number of the detecting aircraft (1 or 2), for the grid-point n . If for some grid-point, both aircraft detect at the same time, then the setting of F and B is arbitrary. In this case F is set to 1 and B to 2, and to identify the case, IE(n) is set to 0.

2.3.2 ε_i , μ_i

In the Single Search Mode an (X, Y) coordinate system is defined as moving with the detecting aircraft. Further, the angles μ and ε are defined in that system. Here these parameters are generalized, so that μ and ε relate to both aircraft, since either aircraft may be the detecting aircraft. The EXECUTIVE Routine produces a value for ε (see [Ref. 1], Section 5.3) and this value is used to set ε_1' and ε_2' . The angles ε_1' and ε_2' are the same in magnitude -- the angle ε between initial headings (see Figures 2.3-1 and 2.3-2). However, ε_1' is measured from aircraft number 1 to aircraft number 2, while ε_2' is measured the other way around. Thus $\varepsilon_1' = -\varepsilon_2'$. The angle μ is generalized. Let μ_1 be the angle between the velocity vector of aircraft 1, $\vec{V}_1$, and the relative velocity vector, $\vec{V}^*$, (see Figure 2.3-1). Let μ_2 be the angle between $\vec{V}_2$ and $\vec{V}^*$ (see Figure 2.3-2). In general, $|\mu_1| \neq |\mu_2|$.

2.3.3 r_k , ρ_k

In Double Search, detection is possible on either of an aircraft's sensory patterns -- optical or radar. Four sets of parameters are required to define the four patterns, two for each aircraft. Thus, whereas the range of the detection pattern, r , and the half-angle, ρ , describe the single detection pattern of importance in the Single Search Mode, here r_k and ρ_k , for $k = 1, 2, 3$ and 4, are required.

2.3.4 $\bar{R}_k$, $\bar{\alpha}_k$

Since detection may occur on one of four patterns, there are four possible positions at the time of detection. In the previous mode the point of detection, defined by the range, R , and the fighter's tracking angle, α_F , lead to the definition of the initial relative parameters. In the current mode the analogous parameters $\bar{R}_k$ and $\bar{\alpha}_k$, for $k = 1, 2, 3, 4$, define the point of detection on the k^{th} pattern in exactly the same way (see Figure 2.3-3).

2.3.5 $Y_{max}(k)$, $Y_{min}(k)$, $Y_B(k)$, $Y_C(k)$

In the Single Search Mode Y_{max} , Y_{min} , Y_B , and Y_C are control parameters uniquely defined by the initial velocities and the detection pattern of the fighter (see Section 5.4 of [Ref. 1]). In the Double Search Mode $Y_{max}(k)$, $Y_{min}(k)$, $Y_B(k)$, and $Y_C(k)$ are these same parameters respectively, defined for the k^{th} pattern (for $Y_{max}(k)$, $Y_{min}(k)$ see Figure 2.3-1 and 2.3-2). They are required to find the point of detection on the k^{th} pattern.

2.3.6 Y^* , Y_{SUP} , Y_{GLB}

In Single Search Y^* is the segment of the projection of the detection pattern (r, ρ) onto the Y-axis. Here there are four patterns to deal with. Consequently, Y^* is generalized to be the superposition of the projections

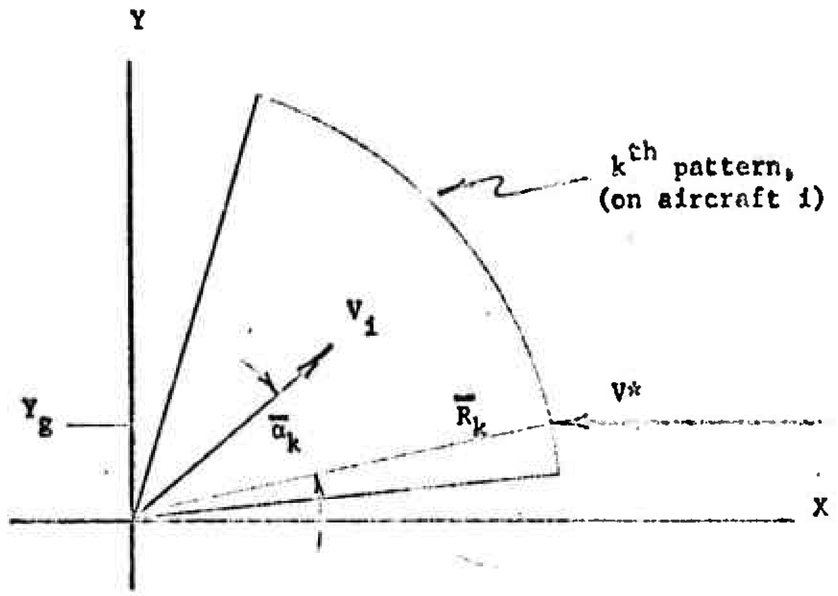


Figure 2.3-3 Detection on the k^{th} Pattern

of the four detection patterns of r_k and ρ_k onto the Y-axis.

As in the Single Search Mode the segment Y^* represents the limits of the Y values within which detection will occur. The limits of the new Y^* are Y_{SUP} and Y_{GLB}, where

$$Y_{SUP} = \max_k \left\{ Y_{max}(k) \right\} \quad ^{1)}$$

and

$$Y_{GLB} = \min_k \left\{ Y_{min}(k) \right\} \quad ^{1)} \quad .$$

The new Y^* will contain overlaps of individual patterns. These represent the regions in which detection can occur on several patterns.

This covers the generalization of the detection parameters.

2.4 Extension of the Single Search Detection in the EM

The method of detection in the Double Search Mode incorporates the procedure by which the point of detection is found on one pattern in the Single Search Mode (see [Ref. 1], Sections 4.2, 5.4, and 5.5.1). This procedure is followed four times. The only concept not covered is the selection of that point which occurs first in time -- the point from which to initialize the engagement. First suppose the detection patterns of the two aircraft are circles. Then clearly the circle with the largest radius will always detect first. More completely, suppose as in Figures 2.3-1 and 2.3-2, $\vec{V}^*$ is directed to the left and the two aircraft are alternately placed at the origin. Then the pattern with the largest X coordinate

---

1) Note the different meanings of max and min within each equation -- one being a functional notation, the other a subscript.

at Y_g will detect first -- in this special case of circles the aircraft with the largest radius -- since decreasing X represents the movement of $\vec{V}^*$ in time. The procedure of placing both aircraft alternately at the "relative origin" and choosing the largest X coordinate is equally valid with partially circular coverage.

If, among the four points of detection only one occurs first, the other three become unimportant, since at this point the aircraft starts to maneuver. It is the point of the four with the largest X value that occurs first. Finally, the X values of all points are found by resolving the $\bar{R}_k$, $\bar{\alpha}_k$ relative coordinates. The magnitude of X is $\sqrt{\bar{R}_k^2 - Y_g^2}$, since the range, $\bar{R}_k$, is from the origin. To find whether X is positive or negative, consider that the angle μ_i, is measured from the positive X axis to the velocity vector $\vec{V}_i$, and the tracking angle α_i, is measured from the line of sight to $\vec{V}_i$. Thus, $|\mu_i - \bar{\alpha}_k|$ is the angular separation of the positive X axis from the line of sight. If $|\mu_i - \bar{\alpha}_k| > \pi/2$, then X is negative, otherwise it is positive.

If the same aircraft detects visually and by search radar simultaneously it is immaterial. However, if both aircraft detect simultaneously, then this must be noted by properly setting the value of JE(n) (see Section 2.3.1), for probabilities later evaluated. Nevertheless the aircraft to be designated the fighter during the engagement, is immaterial.

2.5 The DPM of Double Search

The purpose of the DPM and its method of execution are the same as in the Single Search Mode of ATAC-2. Some changes are required in specifics to accommodate the fact that in the Double Search Mode either aircraft may detect and thereby initiate an engagement. To accomplish the changes entails redefinition of some of the conditional kill probabilities and elimination of others.

2.5.1 The Probabilities of an Engagement and of an Encounter

In Appendix A, Volume IV of [Ref. 1] the probability of detection of the bomber by the fighter is derived. The generalization to the probability of an engagement -- detection by either aircraft -- is obtained in an analogous manner. The segment Y^* is redefined for the Double Search Mode to include the normal projections of both aircraft's detection patterns. In time t this segment sweeps out an area Y^*V^*t in relative space. Detection will take place, resulting in an engagement, if and only if one of the two aircraft is in this area at some time before t. The probability of an engagement as a function of ε, $P_E(\varepsilon)$, has the same form as in the Single Search Mode:

$$P_E(\varepsilon) = 1 - e^{-F\delta D} \qquad (2.5\text{-}1)$$

$$F = Y^*V^*/V_B \quad . \qquad (2.5\text{-}2)$$

Arbitrarily, this ε is used for ε_1'. Due to the symmetry of the initial geometry, the choice of $\varepsilon = \varepsilon_1'$ or $\varepsilon = \varepsilon_2'$ is immaterial. The segment Y^* is generalized to account for the double search in Section 2.3.6.

If P_E is the probability of an encounter, then as in equation (6.3-4), Volume II of [Ref. 1],

$$P_E = \frac{\sum\limits_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_E(\varepsilon) + .5 \sum\limits_{\varepsilon_1, \varepsilon_2} P_E(\varepsilon)}{N_\varepsilon - 1} \qquad (2.5\text{-}3)$$

2.5.2 The Probabilities of Kill

In the Double Search Mode the event of awareness by either aircraft is of no interest. Further, the conditional event of a duel replaces the conditional event of detection of the bomber by the fighter. Let $P_j^1(\varepsilon)$ be the probability that aircraft j is killed, given an engagement at ε. Let $P_j(\varepsilon, n)$ be the probability aircraft j is killed given the engagement defined by ε and n. Thus

$$P_j^1(\varepsilon) = \frac{\sum\limits_{n=1}^{N} P_j(\varepsilon, n)}{N}, \quad j = 1, 2 . \qquad (2.5\text{-}4)$$

Let PK_j^1 be the probability that aircraft j is killed, given an encounter. Then

$$PK_j^1 = \frac{\sum\limits_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_E(\varepsilon)\, P_j^1(\varepsilon) + .5 \sum\limits_{\varepsilon_1, \varepsilon_2} P_E(\varepsilon)\, P_j^1(\varepsilon)}{(N_\varepsilon - 1)\, P_E}, \qquad (2.5\text{-}5)$$

for $j = 1, 2$. These are the basic kill probabilities conditional on an engagement or an encounter.

Further information is available from the EM. One might ask how well an aircraft does when it detects first. Indeed, how likely does it detect the enemy first, given a duel occurs. This latter question relates to the aircraft's avionics capability against a particular enemy. Both questions call for the introduction of additional probabilities. Let $P_D(j, \epsilon)$ be the probability that aircraft j detects its enemy initially, given an engagement at the indicated ϵ. An estimate of $P_D(j, \epsilon)$ is the number of grid-points at which j detected its enemy first or simultaneously, divided by the total number of grid-points "sampled." Thus

$$P_D(j, \epsilon) = N_j(\epsilon)/N \quad , \quad j = 1, 2 \quad , \qquad (2.5\text{-}6)$$

where $N_j(\epsilon)$ is the number of grid-points in which j detected initially for that ϵ. Let $P_D(j)$ be the probability that aircraft j detects its enemy initially, given an encounter. $P_D(j)$ is obtained by averaging the $P_D(j, \epsilon)$ values weighted by the probability of an engagement, divided by the probability of an encounter.

$$P_D(j) = \frac{\sum\limits_{\epsilon \neq \epsilon_1, \epsilon_2} P_D(j, \epsilon)\ P_E(\epsilon) + .5 \sum\limits_{\epsilon_1, \epsilon_2} P_D(j, \epsilon)\ P_E(\epsilon)}{(N_\epsilon - 1)\ P_E} \quad , \qquad (2.5\text{-}7)$$

for $j = 1, 2$.

Let $P_j^2(\epsilon)$ be the probability that aircraft j is killed, given an engagement at ϵ and that aircraft j initially detects its enemy. The computation is made by summing over only those cases in which j detects its enemy. Thus,

$$P_j^2(\epsilon) = \frac{\sum\limits_{n} P_j(\epsilon, n)}{N_j(\epsilon)} \quad , \quad j = 1, 2 \quad . \qquad (2.5\text{-}8)$$

Let PK_j^2 be the probability that aircraft j is killed, given j detects initially. Then

$$PK_j^2 = \frac{\sum\limits_{\varepsilon \neq \varepsilon_1, \varepsilon_2} P_D(j, \varepsilon) P_E(\varepsilon) P_j^2(\varepsilon) + .5 \sum\limits_{\varepsilon_1, \varepsilon_2} P_D(j, \varepsilon) P_E(\varepsilon) P_j^2(\varepsilon)}{(N_\varepsilon - 1)\, P_D(j)\, P_E} \qquad (2.5\text{-}9)$$

for $j = 1, 2$.

SECTION 3

COMPUTATIONAL CHANGES FOR THE DOUBLE SEARCH MODE

3.1 ENGAGEMENT Model Changes

Since the Double and Single Search Modes of ATAC-2 are identical in intra-engagement tactics the only routines in the ENGAGEMENT Model that require changing are those of GRID PREP and GRID (see Section 4.1). The changes necessary are to compute the parameters for the four detection patterns. Instead of computing one set of Y_B, Y_C, Y_{max} and Y_{min}, the Double Search program computes these parameters for four values of k in GRID PREP. The method of computing them for any fixed pattern is the same as in [Ref. 1]. Also, the GRID routine is executed four times to compute $\bar{R}_k$ and $\bar{\alpha}_k$ for the relative geometry (along with ε) of the point of detection on the k^{th} pattern. One singularity occurs here. If Y_g is not between $Y_{max}(k)$ and $Y_{min}(k)$, then detection may not occur on the k^{th} pattern for that ε and grid-point. In such a case $\bar{R}_k$ and $\bar{\alpha}_k$ are set to zero. Finally the relevant point of first detection is found as that point with the largest X coordinate. This X coordinate is easily found, for $\left\{ [\bar{R}_k]^2 - Y_g^2 \right\}^{1/2}$ is its absolute magnitude and the sign of it is given by the sign of the quantity $\pi/2 - | \mu_1 - \bar{\alpha}_k |$. Next the point at detection is found for each aircraft. Then the detecting aircraft is determined, and the value of F is set to that aircraft number, while B is set to the other number, that of the detected aircraft. Finally the code $IE(n)$ is set to identify which aircraft was first to detect in this, the n^{th} grid-point.

3.2 DPM Changes

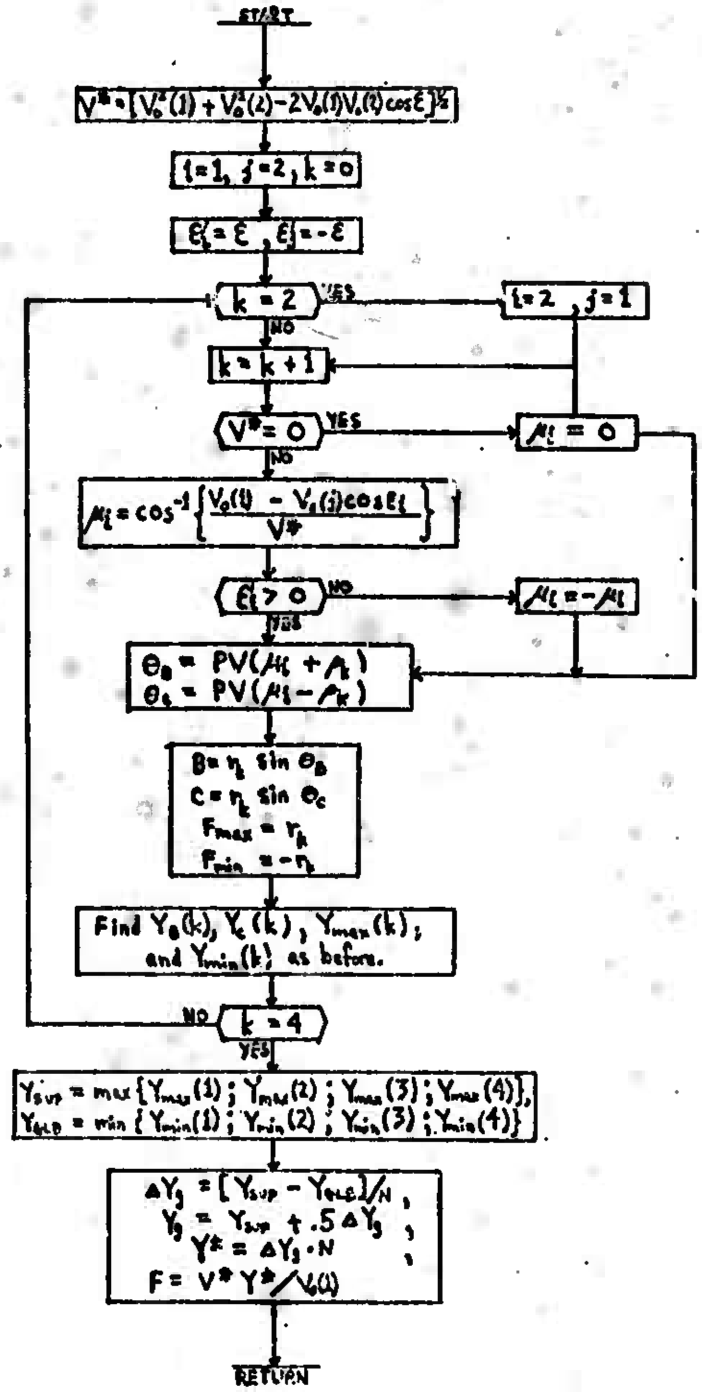
These changes correspond to the discussion given in Section 2.5 above.

SECTION 4

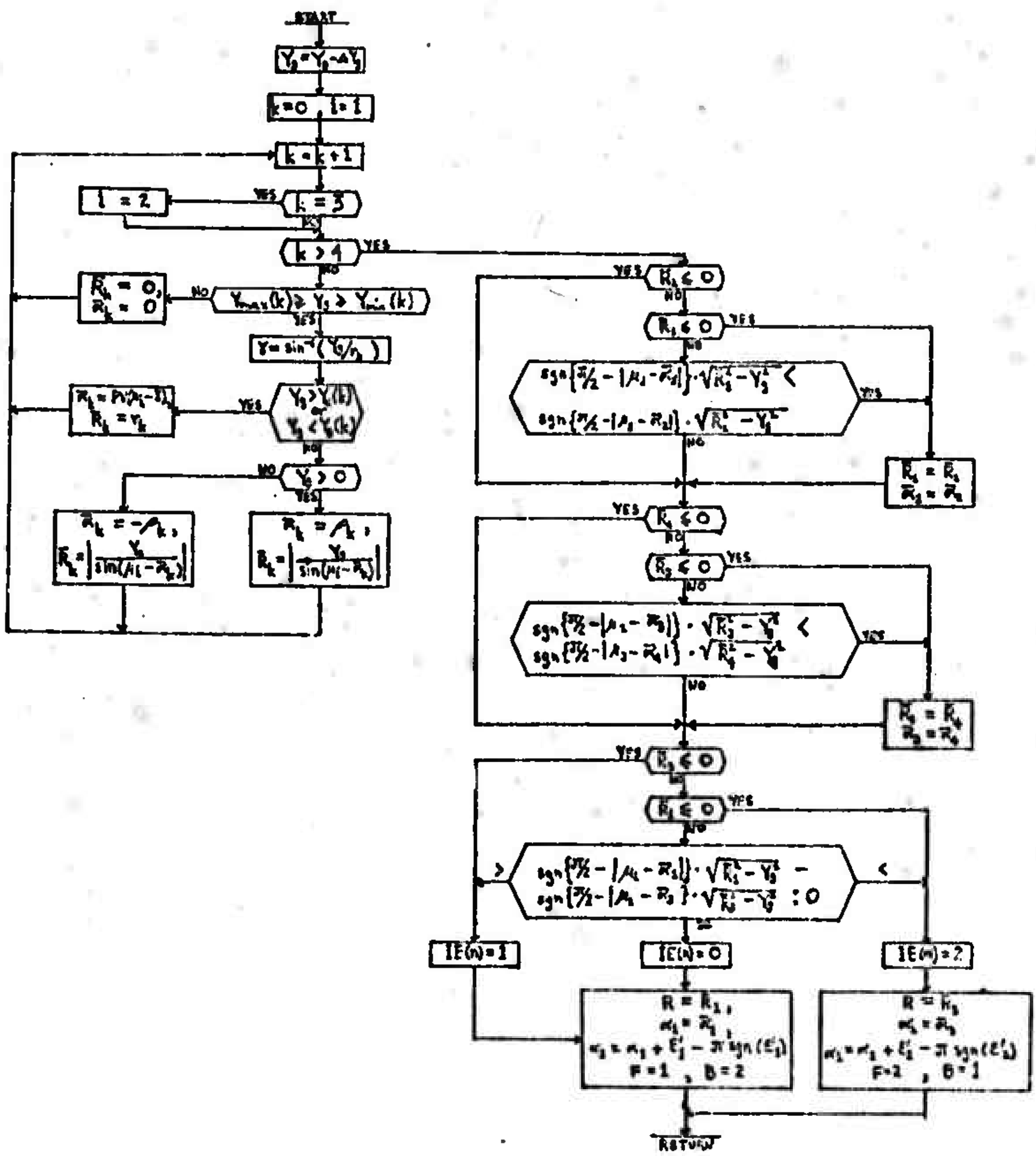
FLOW CHARTS

Only the flow charts of the routines requiring change are presented. The flow chart conventions here are the same as used in Reference [1].

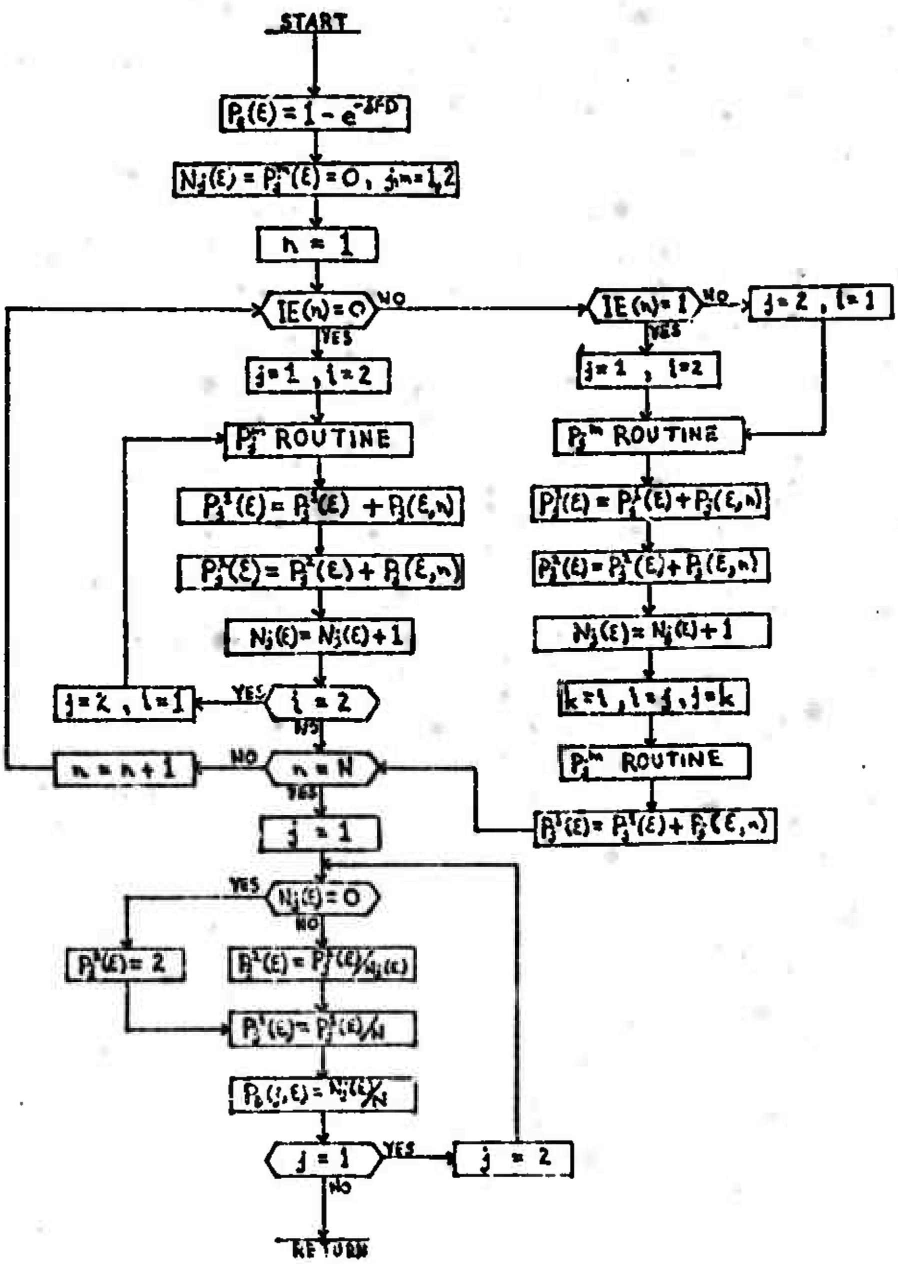
THE GRID PREP



THE GRID OF F vs F



THE CALC. & DATA



THE CALC. UNCOND. DATA

START

$$P_E = \frac{\sum_{\epsilon \neq \epsilon_1, \epsilon_2} P_E(\epsilon) + .5 \sum_{\epsilon_1, \epsilon_2} P_E(\epsilon)}{N_\epsilon - 1}$$

$$P_D(i) = \frac{\sum_{\epsilon \neq \epsilon_1, \epsilon_2} P_E(\epsilon) P_D(i,\epsilon) + .5 \sum_{\epsilon_1, \epsilon_2} P_E(\epsilon) P_D(i,\epsilon)}{(N_\epsilon - 1)\, P_E}$$

$$PK_j^1 = \frac{\sum_{\epsilon \neq \epsilon_1, \epsilon_2} P_E(\epsilon) P_j^1(\epsilon) + .5 \sum_{\epsilon_1, \epsilon_2} P_E(\epsilon) P_j^1(\epsilon)}{(N_\epsilon - 1) P_E} \qquad j = 1, 2$$

$$PK_j^2 = \frac{\sum_{\epsilon \neq \epsilon_1, \epsilon_2} P_E(\epsilon) P_D(j,\epsilon) P_j^2(\epsilon) + .5 \sum_{\epsilon_1, \epsilon_2} P_E(\epsilon) P_D(j,\epsilon) P_j^2(\epsilon)}{(N_\epsilon - 1) P_D(j)\, P_E}$$

RETURN

SECTION 5

PROGRAM LISTING

In this section the program listing of the changed routines is shown.

```
$IBFTC SUB1     XR7,DECK
      SUBROUTINE GRIDPR
      COMMON/INPUTV/TMAX,TMIN,TSTAR,VA,
     1DELEPS,DELTAT,EPSLON,SIGB,GT(3),RMIN,H,
     *N,NM(2),IPS(2),NID,
     1V1(2),V2(2),V3(2),V4(2),W(2),TSTARS(2),
     2ADEC(2),RDET(2),RIFF(2),ROPT(2),
     3RPAS(2),RTRK(2),VC(2),VZ(2),RSTAR(2),ALPDET(2),
     4ALPIFF(2),ALPOPT(2),ALPPAS(2),ALPTRK(2),VMAX(2),GP(2),ACC(2),
     5RF1T(5,15,6,2),RF2T(5,15,6,2),RF3T(5,15,6,2),RF1PT(5,15,6,2),
     6RF2PT(5,15,6,2),RF3PT(5,15,6,2),XID(72),GMIS(6,2),VATAB(10),
     7ALPMIS(6,2),GMAXT(27,2),SIGTAB(15),NVA,NSIGMA,VATAG(27,2),
     8NVAG(2),RC(27,8,2),GBIG(27,2),TC(2),GOFRC(8,2),
     9NUGRC(2)
      COMMON/EXEC/RSMALL(2),RHO(4),IFTR,IBMR,FM(2),BSMALL(2),
      COMMON/GRIDP/VSTAR,FMU(2),YB(4),YC(4),YMAX(4),YMIN(4),
     *DELTYG,YG,F,IGRIDP,IE(10),EPSSUB(2)
      COMMON /COMP/ ICOMP
      COMMON/GRIDSO/GAMMA,ALPHA(2),R
      COMMON/IFORI/RBAR(4),ALPBAR(4),RK(4)
      VSTAR=SQRT(VZ(1)**2+VZ(2)**2-2.*VZ(1)*VZ(2)*COS(EPSLON))
      I=1
      J=2
      K=0
      EPSSUB(I)=EPSLON
      EPSSUB(J)=-EPSLON
    1 CONTINUE
      IF(K.NE.2) GO TO 2
      I=2
      J=1
    2 K=K+1
      IF (VSTAR.EQ.0.) GO TO 10
      FMU(I)=ACOS((VZ(I)-VZ(J)*COS(EPSSUB(I)))/VSTAR)
      IF (EPSSUB(I).LE.0.) FMU(I)=-FMU(I)
      GO TO 11
   10 FMU(I)=0.
   11 CONTINUE
```

```
      THETB=PV(FMU(I)+RHO(K))
      THETC=PV(FMU(I)-RHO(K))
      B=RK(K)*XSIN(THETB)
      C=RK(K)*XSIN(THETC)
      FMAX=RK(K)
      FMIN=-FMAX
      QB=Q(THETB)
      QC=Q(THETC)
      IF(QB.EQ.4.) GO TO 250
      IF(QB.EQ.3.) GO TO 240
      IF(QB.EQ.2.) GO TO 230
      IF(QC.NE.1.) GO TO 200
      IF(THETC.GE.THETB) GO TO 260
      FMAX=B
      FMIN=0.
      B=0.
      GO TO 260
  200 IF(QC.NE.2.) GO TO 205
      IF(B.GT.C) GO TO 201
      FMAX=C
      GO TO 260
  201 FMAX=B
      GO TO 260
  205 IF(QC.EQ.3.) GO TO 201
      FMAX=B
      FMIN=C
      GO TO 260
  230 IF(QC.EQ.1.) GO TO 231
      IF(QC.EQ.2.) GO TO 232
      IF(QC.EQ.3.) GO TO 260
      FMIN=C
      GO TO 260
  231 FMIN=0.
      B=0.
      GO TO 260
  232 IF (B.GT.C) GO TO 260
      FMAX=C
```

```
      FMIN=0.
      B=0.
      GO TO 260
240   IF(QC.EQ.1.) GO TO 241
      IF(QC.EQ.2.) GO TO 242
      IF(QC.EQ.3.) GO TO 243
      IF(B.GT.C) GO TO 244
      FMIN=B
      GO TO 260
241   FMIN=B
      GO TO 260
242   FMAX=C
      FMIN=B
      GO TO 260
243   IF(B.GT.C) GO TO 260
      FMAX=0.
      FMIN=B
      C=0.
      GO TO 260
244   FMIN=C
      GO TO 260
250   IF(QC.EQ.1.) GO TO 260
      IF(QC.EQ.2.) GO TO 251
      IF(QC.EQ.3.) GO TO 252
      IF(B.LE.C) GO TO 260
      FMAX=0.
      FMIN=C
      C=0.
      GO TO 260
251   FMAX=C
      GO TO 260
252   FMAX=0.
      C=0.
260   CONTINUE
      YB(K)=B
      YC(K)=C
      YMAX(K)=FMAX
```

```
      YMIN(K)=FMIN
      IF(K.EQ.4) GO TO 300
      GO TO 1
  300 YSUP=AMAX1(YMAX(1),YMAX(2),YMAX(3),YMAX(4))
      YGLB=AMIN1(YMIN(1),YMIN(2),YMIN(3),YMIN(4))
      DELTYG=(YSUP-YGLB)/FLOAT(N)
      YG=YSUP+.5*DELTYG
      YSTAR=DELTYG*FLOAT(N)
      F=(VSTAR*YSTAR)/VZ(1)
      RETURN
      END
```

```
$IBFTC SUB2     XR7,DECK
      SUBROUTINE SOGRID
      COMMON/EXEC/RSMALL(2),RHO(4),IFTR,IBMR,FM(2),BSMALL(2),
      COMMON/GRIDSO/GAMMA,ALPHA(2),R
      COMMON/GRIDP/VSTAR,FMU(2),YB(4),YC(4),YMAX(4),YMIN(4),
     *DELTYG,YG,F,IGRIDP,IE(10),EPSSUB(2)
      COMMON/INPUTV/TMAX,TMIN,TSTAR,VA,
     1DELEPS,DELTAT,EPSLON,SIGB,GT(3),RMIN,H,
     *N,NM(2),IPS(2),NID,
     1V1(2),V2(2),V3(2),V4(2),W(2),TSTARS(2),
     2ADEC(2),RDET(2),RIFF(2),ROPT(2),
     3RPAS(2),RTRK(2),VC(2),VZ(2),RSTAR(2),ALPDET(2),
     4ALPIFF(2),ALPOPT(2),ALPPAS(2),ALPTRK(2),VMAX(2),GP(2),ACC(2),
     5RF1T(5,15,6,2),RF2T(5,15,6,2),RF3T(5,15,6,2),RF1PT(5,15,6,2),
     6RF2PT(5,15,6,2),RF3PT(5,15,6,2),XID(72),GMIS(6,2),VATAB(10),
     7ALPMIS(6,2),GMAXT(27,2),SIGTAB(15),NVA,NSIGMA,VATAG(27,2),
     8NVAG(2),RC(27,8,2),GBIG(27,2),TC(2),GOFRC(8,2),
     9NUGPC(2)
      COMMON /COMP/ ICOMP
      COMMON/IFORI/RBAR(4),ALPBAR(4),RK(4)
      DATA PI/3.14159265/
      DATA TOL/.000001/
      YG=YG-DELTYG
      K=0
      I=1
    1 CONTINUE
      K=K+1
      IF (K.EQ.3) I=2
      IF (K.GT.4) GO TO 20
      IF ((YG.LE.YMAX(K)).AND.(YG.GE.YMIN(K))) GO TO 2
      RBAR(K)=0.
      ALPBAR(K)=0
      GO TO 1
    2 GAMMA=ASIN(YG/RK(K))
      IF ((YG.GT.YC(K)).OR.(YG.LT.YB(K))) GO TO 3
      IF(YG.LE.0.) GO TO 4
      ALPBAR(K)=RHO(K)
```

```
      RBAR(K)=ABS(YG/SIN(FMU(I)-ALPBAR(K)))
      GO TO 1
3     ALPBAR(K)=PV(FMU(I)-GAMMA)
      RBAR(K)=RK(K)
      GO TO 1
4     ALPBAR(K)=-RHO(K)
      RBAR(K)=ABS(YG/SIN(FMU(I)-ALPBAR(K)))
      GO TO 1
20    CONTINUE
      IF ( RBAR (2) .LE. 0.0 ) GO TO  21
      IF ( RBAR (1) .LE. 0.0 ) GO TO  205
      IF ( SGN (-ABS(FMU(1) - ALPBAR(1) ) + PI / 2.0 )   *
     1 SQRT ( ( RBAR(1)**2 ) - YG * YG ) .LT.
     2 SGN ( - ABS( FMU(1) - ALPBAR(2) ) + PI / 2.0 ) *
     3 SQRT ( RBAR(2)**2 - YG*YG )   ) GO TO 205
      GO TO 21
205   CONTINUE
      RBAR(1)=RBAR(2)
      ALPBAR(1)=ALPBAR(2)
21    CONTINUE
      IF ( RBAR (4) .LE. 0.0 ) GO TO  22
      IF ( RBAR (3) .LE. 0.0 ) GO TO  215
      IF ( SGN (-ABS(FMU(2) - ALPBAR(3) ) + PI / 2.0 )   *
     1 SQRT ( ( RBAR(3)**2 ) - YG * YG ) .LT.
     2 SGN ( - ABS( FMU(2) - ALPBAR(4) ) + PI / 2.0 ) *
     3 SQRT ((RBAR(4)**2)- YG*YG ) )  GO TO 215
      GO TO 22
215   CONTINUE
      RBAR(3)=RBAR(4)
      ALPBAR(3)=ALPBAR(4)
22    CONTINUE
      IF ( RBAR (3) .LE. 0.0 ) GO TO  23
      IF ( RBAR (1) .LE. 0.0 ) GO TO  25
      IF ( SGN (-ABS(FMU(1) - ALPBAR(1) ) + PI / 2.0 )   *
     1 SQRT ( ( RBAR(1)**2 ) - YG * YG )
     2 SGN ( - ABS( FMU(2) - ALPBAR(3) ) + PI / 2.0 ) *
     3 SQRT ((RBAR(3)**2)- YG*YG ) )25,24,23
```

```
23    IF(IGRIDP)=1
      GO TO 27
24    IF(IGRIDP)=0
      GO TO 27
25    IF(IGRIDP)=2
26    R=RBAR(3)
      ALPHA(2)   = ALPBAR(3)
      ALPHA(1)=ALPHA(2)+EPSSUB(2)-PI*SGN(EPSSUB(2))
      IFTR=2
      IBMR=1
      RETURN
27    R=RBAR(1)
      ALPHA(1)=ALPBAR(1)
      ALPHA(2)=ALPHA(1)+EPSSUB(1)-PI*SGN(EPSSUB(1))
      IFTR=1
      IBMR=2
      RETURN
      END
```

```
$IBFTC SUB28
      SUBROUTINE EPSCAL
      COMMON/EPSCA/PD(73),PU(73),P(2,2,73),PZ(2,73),IGRIDP
      COMMON/EXEC/NEPS,IFTR,IBMR,TMAXBF
     *,TTMAX
      COMMON/COMP/ICOMP
      COMMON/INPUTC/D,RHO,TC(2),M(2),IWK(6,2),
     1PKP(6,2),N,NRUNC,NDPMID,DPMID(72)
      COMMON/PKGCAL/PKL,PKG,PKE,PCCEPN(73,25),PCCEPS(73)
      COMMON/INPUTT/NRUN,NID,XID(72),NTEPS,EPSLON(73),
     1NN,TSMALL(25),TAWARE(25),F(25),T(6,2,25)
      COMMON/GRIDD/I,J,LIMA,LIMB,LIMC,IU,A,PK(6,2)
      COMMON/PJUC/PP(2,2,73,25), PPZ(2,73,25)
      COMMON/SICLV/TIMSIC(2),SICTIM(2,25),SICEPS(2,5)
      COMMON/IFORI/IE(10),PE(25),NJ(2,25),PEE,PDJ(2,25),PDI(2)
      IU=2
      PE(NEPS)=1.
      DO 1 J=1,2
      NJ(J,NEPS)=0
      DO 1 M12=1,2
    1 P(M12,J,NEPS)=0.
      DO 15 IJ=1,N
      IGRIDP=IJ
      IF (IE(IGRIDP).NE.0) GO TO 10
      J=1
      I=2
    5 CALL PJU
      P(1,J,NEPS)=P(1,J,NEPS)+PP(IU,J,NEPS,IGRIDP)
      P(2,J,NEPS)=P(2,J,NEPS)+PP(IU,J,NEPS,IGRIDP)
      NJ(J,NEPS)=NJ(J,NEPS)+1
      IF (I.NE.2) GO TO 15
      J=2
      I=1
      GO TO 5
   10 IF (IE(IGRIDP).NE.1) GO TO 11
      J=1
      I=2
```

```
      GO TO 12
   11 J=2
      I=1
   12 CALL PJU
      P(1,J,NEPS)=P(1,J,NEPS)+PP(IU,J,NEPS,IGRIDP)
      P(2,J,NEPS)=P(2,J,NEPS)+PP(IU,J,NEPS,IGRIDP)
      NJ(J,NEPS)=NJ(J,NEPS)+1
      KTEMP=I
      I=J
      J=KTEMP
      CALL PJU
      P(1,J,NEPS)=P(1,J,NEPS)+PP(IU,J,NEPS,IGRIDP)
   15 CONTINUE
   20 CONTINUE
      J=1
   21 CONTINUE
      IF (NJ(J,NEPS).EQ.0) GO TO 22
      P(2,J,NEPS)=P(2,J,NEPS)/FLOAT(NJ(J,NEPS))
      GO TO 23
   22 P(2,J,NEPS)=2.
   23 CONTINUE
      P(1,J,NEPS)=P(1,J,NEPS)/FLOAT(N)
      PDJ(J,NEPS)=FLOAT(NJ(J,NEPS))/FLOAT(N)
      IF (J.NE.1) RETURN
      J=2
      GO TO 21
      END
```

```
$IBFTC SUB29
      SUBROUTINE UNCOND
      COMMON/EKT1/EKT,YN,EKF
      COMMON/EXEC/NEPS,IFTR,IBMR,TMAXBF
     *,TTMAX
      COMMON/PKGCAL/PKL,PKG,PKE,PCCEPN(73,25),PCCEPS(73)
      COMMON/INPUTT/NRUN,NID,XID(72),NTEPS,EPSLON(73),
     1NN,TSMALL(25),TAWARE(25),F(25),T(6,2,25)
      COMMON/EPSCA/PD(73),PU(73),P(2,2,73),PZ(2,73),IGRIDP
      COMMON/UNCON/PUPD,PDD,PKZB,PKK(2,2)
     1,PKBU,PKD1FB,PKD2FB,PKJIA(2,2)
      COMMON/SICLV/TIMSIC(2),SICTIM(2,25),SICEPS(2,5)
      COMMON /NUOUT/PKB
      COMMON/NUPROB/PKBGDE(25),PKFGDE(25),PKBGD,PKFGD
      COMMON/IFORI/IE(10),PE(25),NJ(2,25),PEE,PDJ(2,25),PDI(2)
      NEPSM1=NTEPS-1
      XEPS=NEPSM1
      PEE=.5*(PE(1)+PE(NTEPS))
      PDI(1)=.5*(PDJ(1,1)*PE(1)+PDJ(1,NTEPS)*PE(NTEPS))
      PDI(2)=.5*(PDJ(2,1)*PE(1)+PDJ(2,NTEPS)*PE(NTEPS))
      PKK(1,1)=.5*(PE(1)*P(1,1,1)+PE(NTEPS)*P(1,1,NTEPS))
      PKK(2,1)=.5*(PE(1)*P(1,2,1)+PE(NTEPS)*P(1,2,NTEPS))
      PKK(1,2)=.5*(PE(1)*PDJ(1,1)*P(2,1,1)+PE(NTEPS)*PDJ(1,NTEPS)*P(2,1,
     1NTEPS))
      PKK(2,2)=.5*(PE(1)*PDJ(2,1)*P(2,2,1)+PE(NTEPS)*PDJ(2,NTEPS)*P(2,2,
     1NTEPS))
      DO 10 IEPS=2,NEPSM1
      PEE=PEE+PE(IEPS)
      DO 10 M1=1,2
      PDI(M1)=PDI(M1)+PDJ(M1,IEPS)*PE(IEPS)
      PKK(M1,1)=PKK(M1,1)+PE(IEPS)*P(1,M1,IEPS)
      PKK(M1,2)=PKK(M1,2)+PE(IEPS)*PDJ(M1,IEPS)*P(2,M1,IEPS)
   10 CONTINUE
      PEE=PEE/XEPS
      DO 12 M1=1,2
      PDI(M1)=PDI(M1)/(XEPS*PEE)
      PKK(M1,1)=PKK(M1,1)/(XEPS*PEE)
```

```
      PKK(M1,2)=PKK(M1,2)/(XEPS*PDI(M1)*PEF)
   12 CONTINUF
      RETURN
      END
```

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HERMAN WOOD
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13. ABSTRACT

ATAC-2 is a simulation model designed to help evaluate fighters in air-to-air combat. The model treats the one vs. one dogfight which arises from a random search siutation. Both aircraft in the combat are (usually) aggressive. The two principal outputs from the model are the probability a given aircraft is killed in the fight and the expected number of enemy aircraft an aircraft kills over its useful life. Combat is restricted to a fixed altitude. The maneuvers are dynamic in that each aircraft responds to the situation at each moment in a duel depending on the information it has about an opponent's activities. (U)

Inputs include, for each aircraft, search and tracking radar characteristics, passive radar sensors, optical capability, IFF, energy-maneuverability data, weapon loadings, weapon characteristics, and weapon kill probabilities. (U)

The rationale for the model specifics are presented. Flow charts and program listings are included. The model has been run repeatedly on an IBM 7094. (U)

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